Wireless Multihop Networks: Performance Analysis

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May 4, 2000

Outline

- Definition
- Literature survey
  - Prior studies on TDMA
  - Works on CDMA
- Single cell with fixed repeaters
  - Approach
  - Results
- General structure
  - Scenario
  - Interference calculation
- Issues

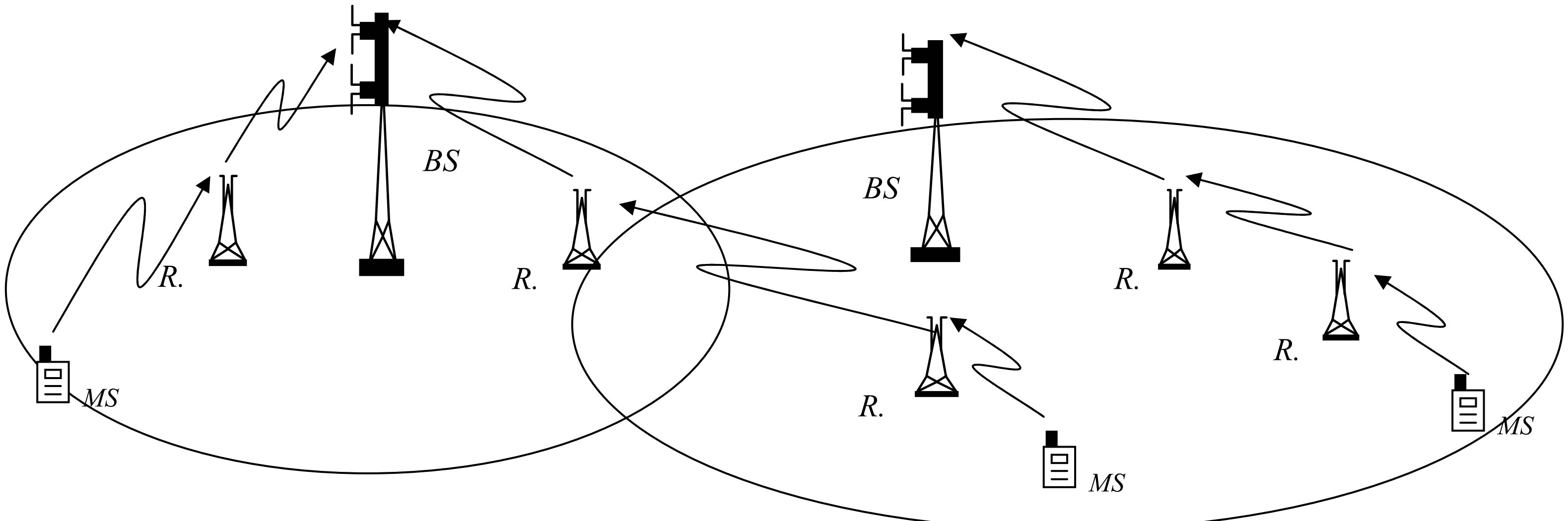
Wireless Ad hoc Networks

- Definition
  - A network with dynamic, random and multihop topology
  - A set of nodes which some of them can act both as a terminal and a router form the network routing infrastructure in an ad_hoc fashion

- Characteristics
  - Dynamic topology which changes randomly with time
  - Bandwidth constrained and variable capacity links
  - Energy constrained
  - Unilateral links

Benefits of Multihop Networking

- By shortening paths, transmitted power is highly reduced
- One other source for power reduction is that the propagation model approximately changes from $1/r^4$ to $1/r^2$ for short links
- Power reduction results in reduced interference
- Routers can be used to distribute the traffic uniformly



Problem Statement

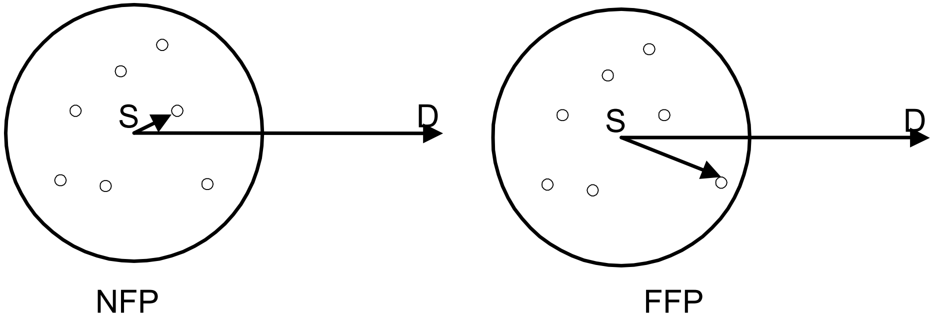
- How to model a network of uniformly distributed base stations, repeaters and terminals?
- Reducing the power consumed in the network, does it necessarily result in higher capacity?
- What are the efficient routing algorithms?
- How to optimize system parameters?

Review of the Literature

- Assumptions in previous research work
  - Nodes are distributed as a two dimensional Poisson process
  - Nodes acting both as terminals generating packets and routers retransmitting packets
  - Channel access protocol is slotted ALOHA
  - Heavy traffic condition, nodes always have a packet to transmit
  - A node in transmit mode can not receive due to its own interference
  - In each slot, nodes try to transmit one packet with the fixed probability *p* regardless of the number of received packets
  - Uniform traffic has been assumed, i.e., every node will transmit to any other node with equal probability

Review of the Literature (cont'd)

- Transmission Strategies
  - What should be the transmission range?
  - To whom should the transmission be addressed?
- Two methods have been investigated in the literature
  - NFP (Nearest with forward progress)
  - FFP (Farthest with forward progress)



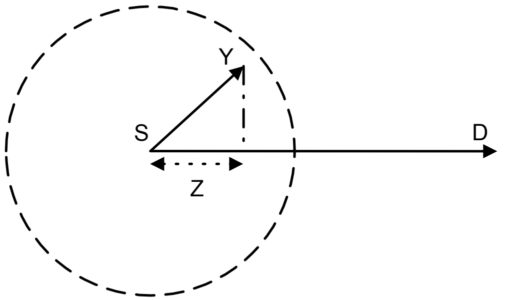
Review of the Literature (cont'd)

- Performance measure
  - Traffic is inversely proportional to the distance packets travel towards the destination in each hop (Progress)

S : Source , D : Destination

Y : Local Destination

Z : Progress



  - Expected Forward Progress (EFP)

EFP = Z * Pr[success S ->Y]

Review of the Literature (cont'd)

- Results
  - TDMA, e.g. Hou and Li (1986)
    - for NFP strategy with transmission range reduced to the local destination

$$N_{opt} = \lambda_t \pi R^2 = 7.7, \qquad p_{opt} = 0.320$$

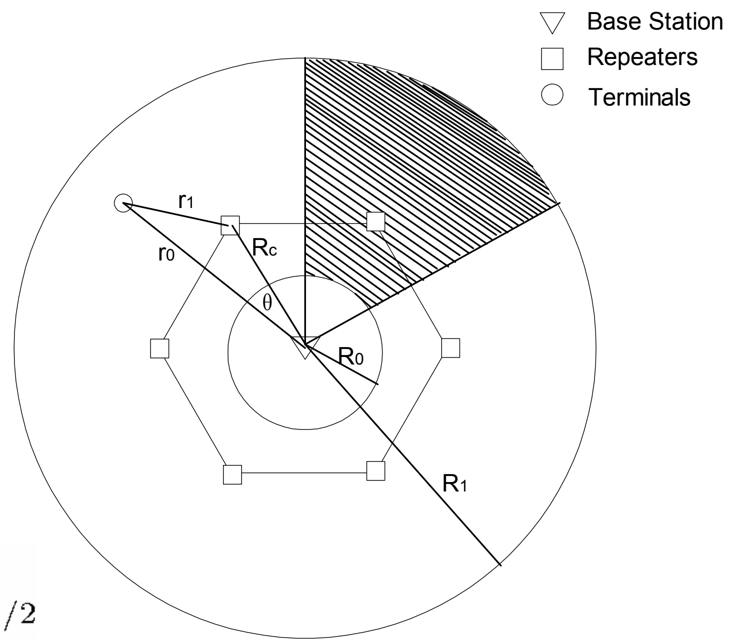
  - CDMA, e.g. Sousa and Silvester (1990)
    - for fixed transmission power and fixed source to local destination distance

$$N_{opt} = \lambda_t \pi R^2 = 1.33\sqrt{K(\mu_c)}, \quad p_{opt} = 0.271$$

$K(\mu_c)$ is the multiuser capability of spread-spectrum

Single Cell with Fixed Repeaters

- Network Structure
  - Base Station
  - Repeaters
  - Terminals



$$I_{1i}(r_0,\theta) = \left(1 + \frac{R_c^2}{r_0^2} - \frac{2R_c\cos\theta}{r_0}\right)^{\alpha/2}$$

Transmission Scenario

- The uplink and downlink are assumed to carry the information in FDD
- Access technique is CDMA
- Terminals and repeaters transmit packets as slotted ALOHA
- In each frequency band, repeaters receive data in some time slots and retransmit them in the same frequency band but in different time slots
- Each terminal sends and receives on one channel (spreading sequence)
- Repeaters may send and receive on several channels, dynamically allocated by base station.

Transmission Scenario (Cont'd)

- In the uplink, ideal power controlled both at base station and repeaters
- Network structure has circular symmetry, therefore selection region of neighboring repeaters
- Traffic, interference and error rate are affected nonlinearly by selection region of repeaters and base station
- Different structures for defining the regions can be considered
- Layer 0 : Interior region including the base station
- Layer 1 : Exterior region

Throughput Calculation

- Throughput (N_s) : The number of successfully received packets at a receiver at each slot
- N_s is a binomial random variable with parameters :
  - p_s : Probability of a packet successful reception
  - N : Number of packets transmitted to that receiver
- p_s is a function of N and Interference due to transmissions to other receivers, I

$$
\begin{aligned}
\gamma &= E(N_s) = E(E(N_s|N, I)) \\
&= E(N p_s(N, I)) \\
&= \sum_{n=1}^{\infty} n E(p_s(n, I)) \Pr(N = n)
\end{aligned}
$$

Throughput Calculation (Cont'd)

- For the best very long codes

$$p_s(N,I) = \begin{cases} 1 & N + \frac{I}{P_x} < K(\mu_c) \\ 0 & N + \frac{I}{P_x} > K(\mu_c) \end{cases}$$

- Throughput

$$\gamma = \sum_{n=1}^{\lfloor K(\mu_c) \rfloor} n \Pr(I/P_x < K(\mu_c) - n) \Pr(N = n)$$

Interference Calculation

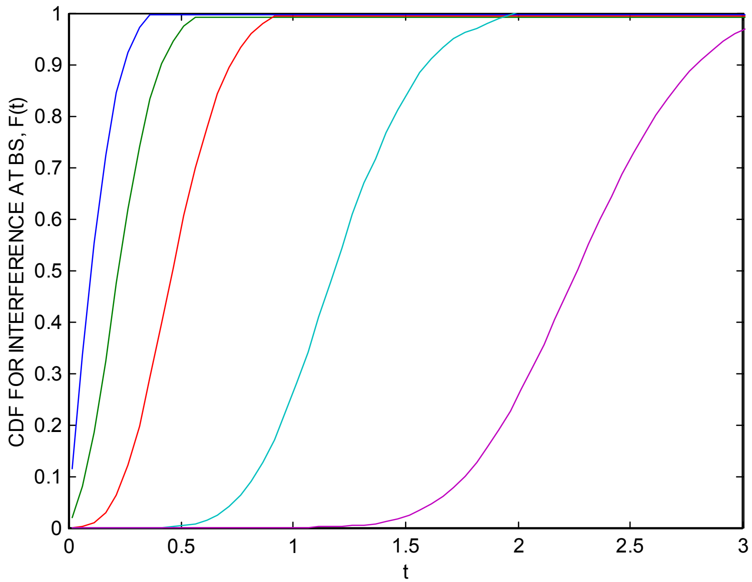
$$\Pr\{I > t\} = \sum_{k=0}^{\infty} e^{-\lambda_I} \frac{{\lambda_I}^k}{k!} \Pr\{I > t | k \text{ in } D_I\}$$

$$\Pr\{I > t | k \text{ in } S_I\} \leq E\{e^{\rho(I-t)} | k \text{ in } S_I\}$$
$$= e^{-\rho t}(E\{e^{\rho I_i}\})^k$$

$$\begin{aligned} \Pr\{I &> t\} \\ &\leq \min_{\rho>0}\{\exp(-\rho t + \lambda_I E\{e^{\rho I_i}\} - \lambda_I)\} \\ &= \exp(\min_{\rho>0}\{-\rho t + \lambda_I E\{e^{\rho I_i}\} - \lambda_I\}). \end{aligned}$$

$$\lambda_I E\{I_i e^{\rho_{\min} I_i}\} = t$$

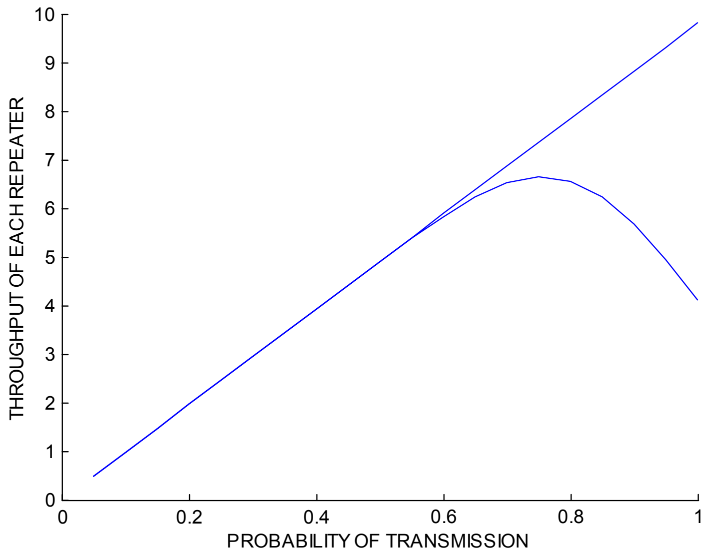
Interference



- Cumulative distribution function for interference at BS due to transmission of all terminals in layer 1. Curves to right are related to higher p_t

$$\lambda_t = 5, R_0 = 2/3, R_1 = 2, p_t = .05, .1, .2, .5, 1$$

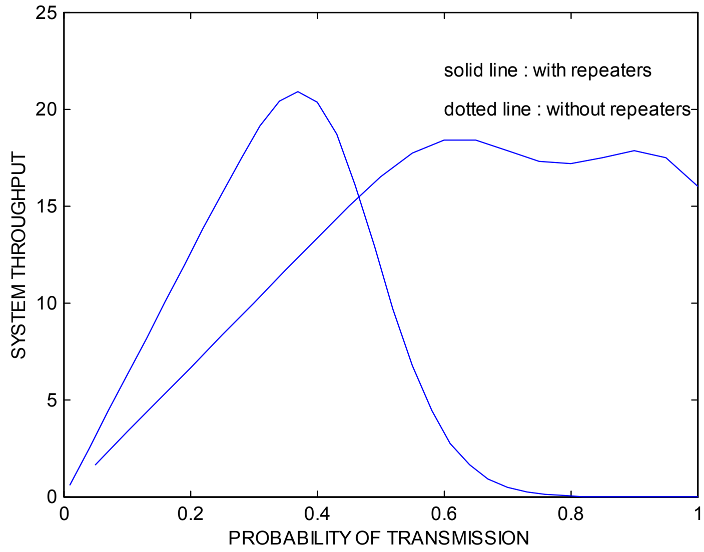
Repeaters Throughput

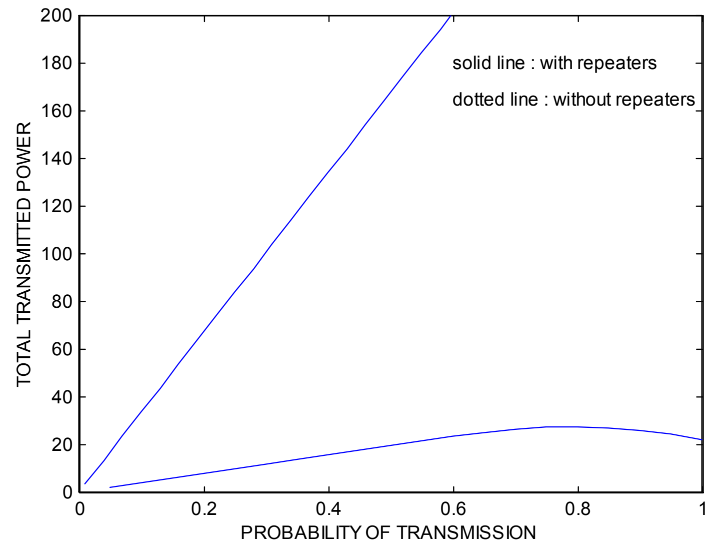


Throughput of each repeater in comparison with the ideal case of $Kc = \infty$.

System parameters are $K_c = 30$, $\lambda_t = 5$, $R_0 = 1/2$, $R_1 = 2$

Comparison for with and without Repeaters

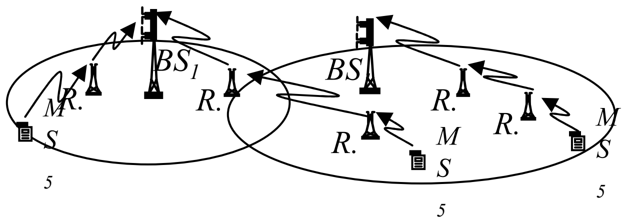


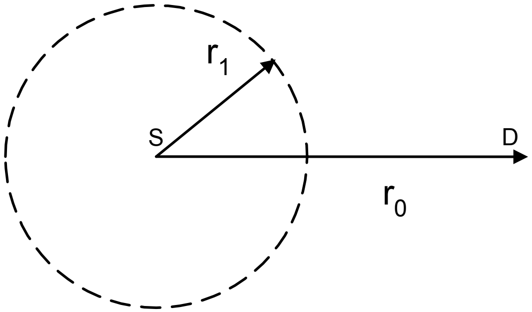


Throughput and Power consumption in the new structure in comparison to the standard case of not using repeaters in the structure

$K_c = 30, \lambda_t = 5, R_0 = 1/2, R_1 = 2$

General Case





- Interference calculation

$$f(r_0, r_1) = f(r_0) f(r_1 | r_0) = \frac{2r_0}{a^2} \begin{cases} 2\pi\lambda r_1 \exp(-\lambda\pi r_1^2) & r_1 < r_0 \\ \exp(-\lambda\pi r_0^2)\delta(r_0 - r_1) & r_1 = r_0 \\ 0 & r_1 > r_0 \end{cases}$$

$$u = r_1/r_0$$

$$w = r_0$$

General Case (Cont'd)

$$f_U(u) = 2(1 - e^{-Nu^2}(1 + Nu^2))/Nu^3 + (1 - e^{-N})\delta(u-1)/N$$

$$f_H(h) \quad = \quad \frac{1}{2}\frac{1 - e^{-N\sqrt{h}}(1 + N\sqrt{h})}{N\left(\sqrt{h}\right)^3} + \frac{1 - e^{-N}}{N}\delta(h-1)$$

$$\begin{aligned}\lim_{N\to\infty} N\int_0^1 h^n f_H(h)dh &= \lim_{N\to\infty}\int_0^1 \frac{1}{2}h^{n-3/2}(1 - e^{-N\sqrt{h}} - e^{-N\sqrt{h}}N\sqrt{h})\,dh + 1 \\ &= \int_0^1 \frac{1}{2}h^{n-3/2}\,dh + 1 = \frac{1}{2n-1} + 1 \qquad \text{for } n > 1/2\end{aligned}$$

General Case (Cont'd)

$$M(s) = E(e^{sH}) = E\left(\sum_{k=0}^{\infty} \frac{(sH)^k}{k!}\right) = \sum_{k=0}^{\infty} \frac{s^k}{k!} E(H^k)$$

$$= 1 + \frac{1}{N} \sum_{k=1}^{\infty} \frac{s^k}{k!} (1 + \frac{1}{(2k-1)}) + O(\frac{1}{N^2}) = \frac{N - i\sqrt{s}\sqrt{\pi}\,\mathrm{erf}\,(i\sqrt{s})}{N} + O(\frac{1}{N^2})$$

$$M_T(\omega) = E(e^{i\omega \sum H}) = \left(E(e^{i\omega H})\right)^N$$

$$= \lim_{N\to\infty} \left(1 - \frac{i}{N}\sqrt{(i\pi\omega)}\,\mathrm{erf}\left(i\sqrt{(i\omega)}\right) + O(\frac{1}{N^2})\right)^N$$

$$= \exp\left((1-i)\sqrt{\pi\omega/2}\,\mathrm{erf}\left(-(1-i)\sqrt{\omega/2}\right)\right) \quad \omega > 0$$

$$M_T(\omega) = M_T^*(-\omega)$$

$$f_{H_T}(h) = 2\,\mathrm{Re}\left\{\int_0^{\infty} \exp\left(-i\omega h + (1-i)\sqrt{\pi\omega/2}\,\mathrm{erf}\left(-(1-i)\sqrt{\omega/2}\right)\right) d\omega\right\}$$

Conclusion

- Throughput calculation gives us a measure that can be easily compared with maximum system capacity
- Models should be developed based on the proposed method for calculating throughput in general case
- Routing algorithms proposed so forth for multihop networking are mostly based on efficient connectivity, further work need to be done to consider the effect of air interference

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