# On the Capacity of Wireless Packet CDMA Multihop Networks

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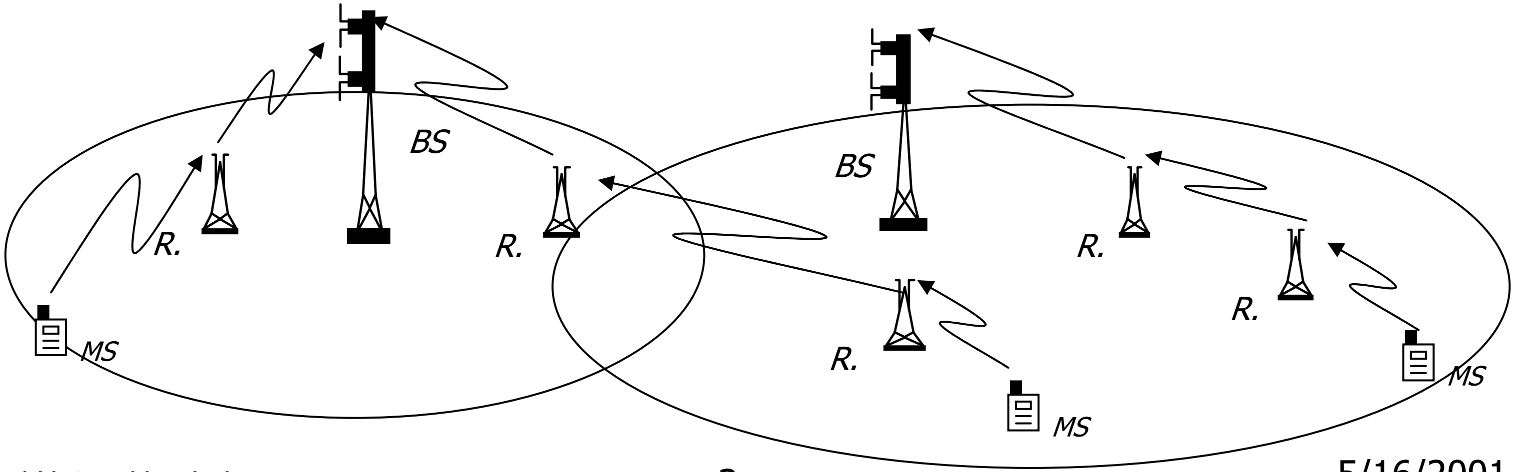
November 14, 2000

# Outline

- Problem statement and motivation
- A simple architecture
  - Structure, radio link model
- Link capacity in single user system
  - Diversity techniques, multi-element arrays, space-time coding
- Link capacity in multiple user system
  - Multiuser detection, smart antennas/interference cancellation
- Network capacity
  - Medium access techniques, Routing strategy
- Performance Analysis
  - Throughput, traffic rate, interference
- Conclusion

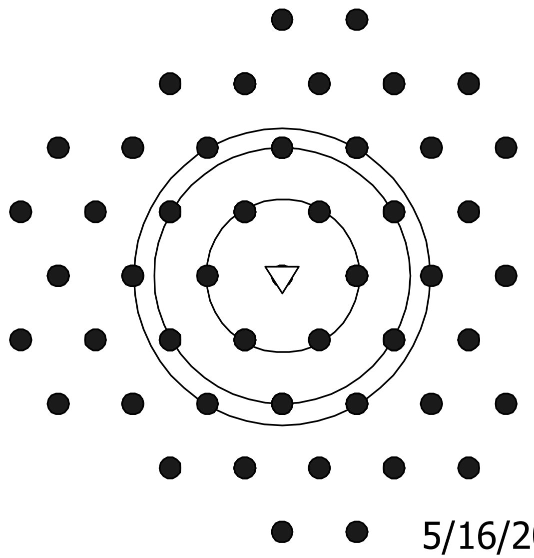
# Benefits of Multihop Networking

- By shortening paths, transmitted power is highly reduced
- One other source for power reduction is that the propagation model approximately changes from $1/r^4$ to $1/r^2$ for short links
- Routers can be used to distribute the traffic uniformly
- Bursty traffic can be handled through a route without affecting the reception capability of the other parts of the network



# Problem Statement

- Users share three main domains to transfer data
  - Time
  - Frequency
  - Space
- Increasing the number of channels in the space:
  - By increasing the number of network cells
  - In very small cells, base stations and their connection to the wired backbone is quite costly



# Problem Statement

- Capacity of multihop network:  the total rate by which information originated by all nodes reaches the final destinations
- How does different techniques affect the capacity?
- Reducing the power consumed in the network, does it necessarily result in higher capacity?
- What are the efficient routing algorithms?
- How to optimize system parameters?
- How to model this network?

# Radio Link Characteristics

- Time varying multipath channel
  - Angular spread: Signal reception from many different directions
  - Time spread: Signal propagation through different paths experiences different delays
  - Frequency spread: Change of scattering environment with time
- Radio channel model:
  - Fading
  - Shadowing
  - Path loss
- Existing power control techniques
  - Approximately mitigate the effect of shadowing and path loss
  - Received power modeled as a Rayleigh distribution

# Link Capacity in Single-User System

- Maximum spectral efficiency(bits/s/Hz) is obtained based on Shannon formula
  - Capacity is related to channel state information (CSI) available at transmitter and receiver
  - A better strategy can be devised to achieve higher throughput when there is more information available about the channel
- For a single transmit/single receive antenna,flat fading channel, and ideal CSI
  - The optimum power strategy is:
    - No transmission when the fading coefficient is below a threshold
    - Transmission with proportionally lower power as the coefficient increases over that threshold
  - When the signal is highly faded, no power is wasted by transmitting data

# Diversity Techniques

- Multipath fading:
  - Severely attenuates the transmitted signal
  - Provides several replicas of the signal at the receiver
- Main diversity domains:
  - Temporal diversity:
    - Error correcting codes in conjunction with time interleaving
  - Frequency diversity:
    - Equalizers in TDMA or RAKE receivers in CDMA
  - Space diversity:
    - Spatially separated or differently polarized antennas
  - Receiving several replicas of the same packet in different time slots and from different nodes
- Total diversity order: the product of all diversity orders in independent domains
- By increasing diversity order, distribution of fading tends to follow a Gaussian distribution which corresponds to higher capacity limit

# Multiple-in Multiple-out Radio Channels

- High data rate transmission in bandwidth limited channel
  - Using highly spectral efficient code with very large constellation size
  - Increasing the number of channels
- Capacity versus outage:
  - When $T << T_{coh}$ channel capacity is viewed as a random variable since it depends on the instantaneous random channel parameters

    T: transmission duration of a data block

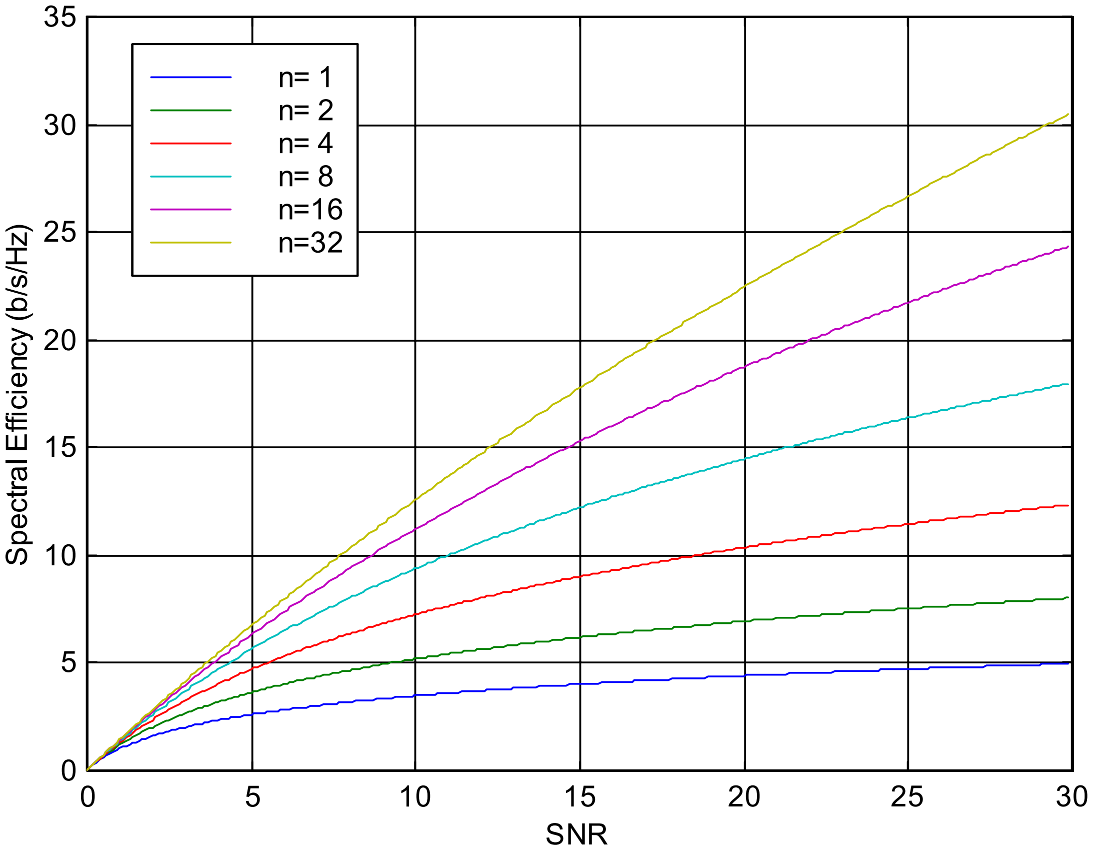
    $T_{coh}$: channel coherence time
- A flat slowly fading channel with $n_R$ receive, $n_T$ transmit antennas
  - Channel can be described by an $(n_R, n_T)$ matrix H which has the propagation information from each transmitter to each receiver
  - Shannon capacity as a random variable (perfect CSI at receiver)

$$C = \log_2\left(\det[I_{n_R} + HH^H \, P_0/n_T P_n]\right)$$

$P_0$: transmit signal power $P_n$: average power of AWGN

# MIMO Radio Channels (Cont'd)

- This method can create up to $n=\min(n_R, n_T)$ parallel channels



# MIMO Radio Channels (Cont'd)

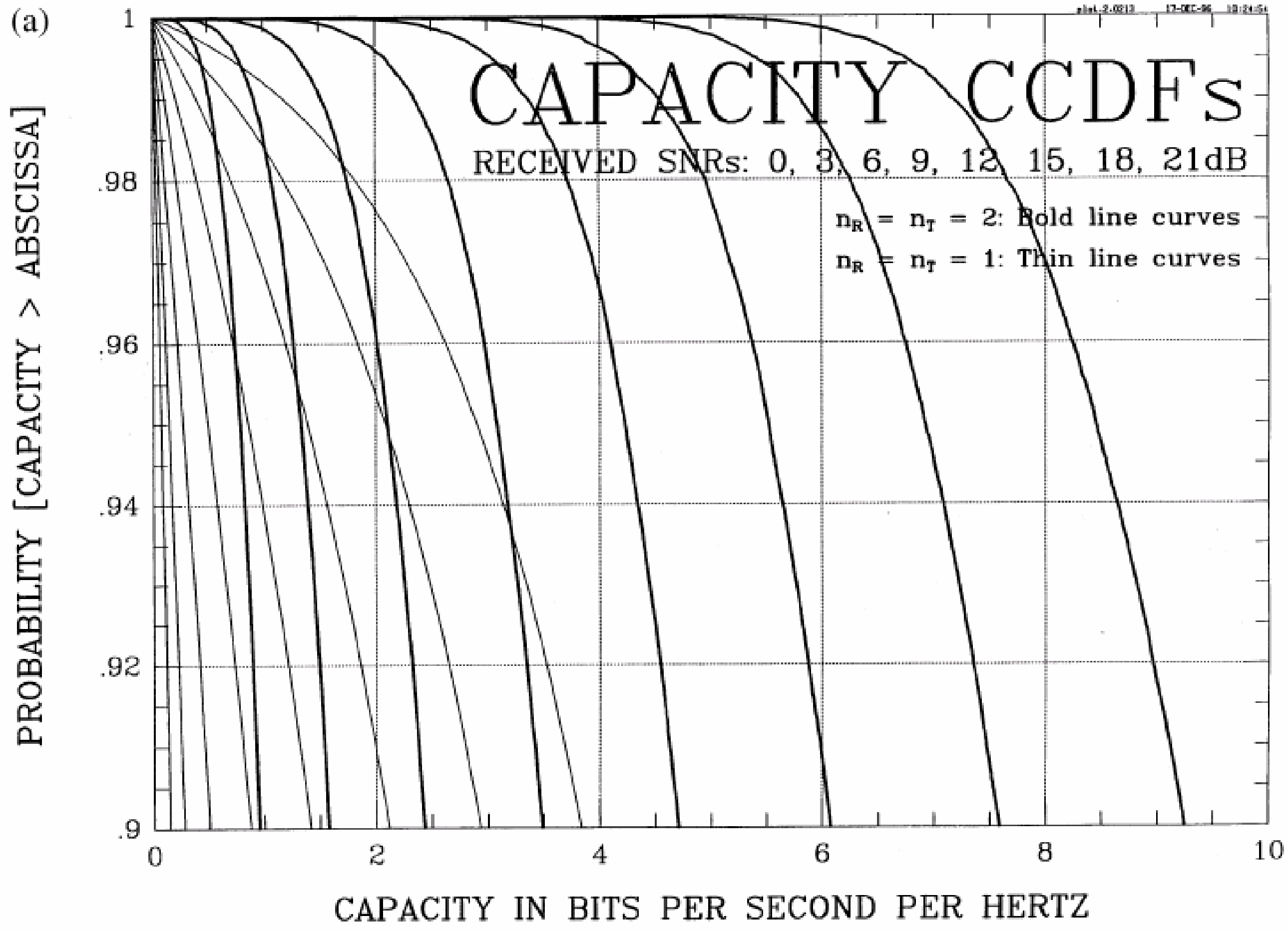
As an example, consider the case of one transmit and multiple receive antennas

$$C = \log_2\left(1 + \frac{P_0}{P_n}\sum_{i=0}^{n}\left|H_i\right|^2\right)$$

- Which can be achieved by a linear beam-former

- The increased capacity in this link is due to two effects.
  - By increasing the number of channels, mean capacity is increased almost linearly with the number of antenna elements
  - By providing temporal, transmit, and receive diversity channel reliability will be highly improved for higher data rates

# MIMO Radio Channels (Cont'd)



# Space-Time Coding

- Uncorrelated diversity branches can be obtained
  - At mobile station: spacing the antenna elements about a half wavelength apart
  - For base stations: an antenna separation of about 20 wavelength is required to obtain a correlation of about 0.7 , with this high value of correlation, a significant diversity improvement can be realized.
- Adding multiple antennas for small devices is expensive and sometimes not practical
- Routers which are normally mounted on bigger devices with better power supplies can afford good space time codes
- Connection of routers together and to the base stations using highly efficient codes is like connecting them by wire
- Outage capacity of 40 b/s/Hz has been reported
  - 95% availability
  - signal to noise ratio of 10 dB
  - eight elements at both transmitter and receiver

# Link Capacity in Multiple User System

- In a conventional cellular network:
  - Increasing the power of all nodes does not affect the signal to interference ratio (for high SNR)
  - When all the information about other transmitting users is limited to the total received interference, power scaling does not affect the performance of the network
  - Link capacity is limited by increasing SNR
- Higher capacity limit is achieved:
  - Using information like spreading codes, direction of signal arrival of other transmitting users
  - This is at the expense of using more complex techniques for decoding

# Multiuser Detection

- When K users are transmitting to a receiver simultaneously, in a flat slowly fading environment, a channel can be described by K random variables $\nu_i$ modeling the propagation from the ith user to the receiver
- For the case that the receiver has perfect CSI and the transmit power of each user is P, the maximum achievable capacity for all users can be obtained as

$$C = \log_2\left(1 + \sum_{i=0}^{K} P_i \upsilon_i \Big/ P_n\right)$$

- For a large number of users

$$C = \log_2\left(1 + KP_{av} / P_n\right)$$

- This capacity can be achieved in CDMA systems where the deleterious effect of fading is mitigated by averaging inter-user effect

# Multiuser Detection

- By increasing the number of transmitted packets, the maximum information that each packet can bear is reduced
- One effective method to overcome this problem is to use methods like Hybrid ARQ
  - Packets are saved at the destination and more parity bits are requested by receiver until it can decode the data
  - By this method, multirate traffic is generated which adaptively changes as a function of channel state
- Multiuser detection
  - Reduces sensitivity of decoding to near far effect and power control
  - In the uplink results to lower bit error rate
  - In the downlink results to high power conservation

# Smart Antennas/Interference Cancellation

- Even with multiuser detection, link capacity will not linearly increase by the number of transmitted packets
- Specially for high signal to noise ratios, splitting power on different channels results in a better performance.
- Cell sectorization using directional antennas at base stations
  - Increase the system capacity in the uplink and downlink by a factor equal to the number of sectors
  - Due to the imperfection in practical antennas, the reliability of reception from terminals located in handoff regions is reduced
  - Due to wide angular spread of the signal at mobiles and routers, receive antennas can not distinguish between direction of signal arrivals from different transmitters
- An M element smart antennas, however, can null M-1 interferers independent of the multi-path environment

# Smart Antennas/Int. Cancellation(Cont'd)

- By adding routers, as the maximum number of nodes communicating to any node is reduced, smart antennas can be more effective
- Another problem arises when space-time coding and smart antenna technique are used together
  - In each symbol period, the set of symbols transmitted on different branches of antenna construct the array weight vector and define the radiation pattern
  - The transmit antenna pattern steers in different directions from symbol to symbol creating interference for other part of the network
  - For reception, this pattern change results in receiving interference from other sources

# Network Capacity

- Space, time, and frequency are the three main domains that must be shared between different network nodes
- Time scheduling is referred to as medium access protocol
- Space sharing is done based on network topology and is usually covered under the topic of routing strategy
- Two main questions:
  - How to use the System State Information (SSI) to define the strategy
  - How to obtain this information

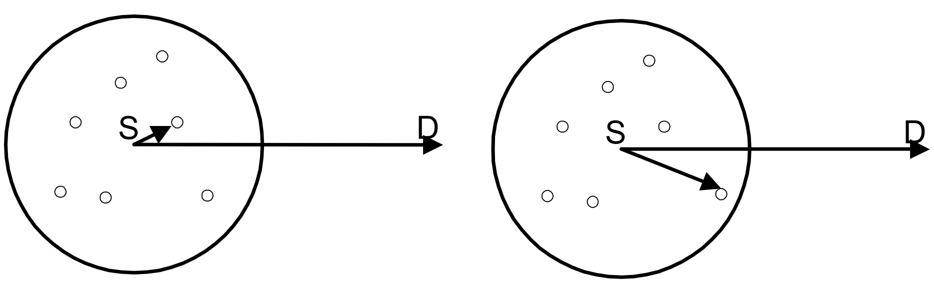
# Medium Access Techniques

- The statistical properties of today's network traffic shows a long range dependency in time and self similarity characteristics
- Just like the single channel case, feeding back system state information (SSI) to the transmitting nodes has a direct effect on network capacity
- Medium access control (MAC) is to guarantee a traffic with
  - Low variance
  - Average close to capacity limit

  - Reduce the probability of outage
  - Increase system throughput

# Impact of Routing on Capacity

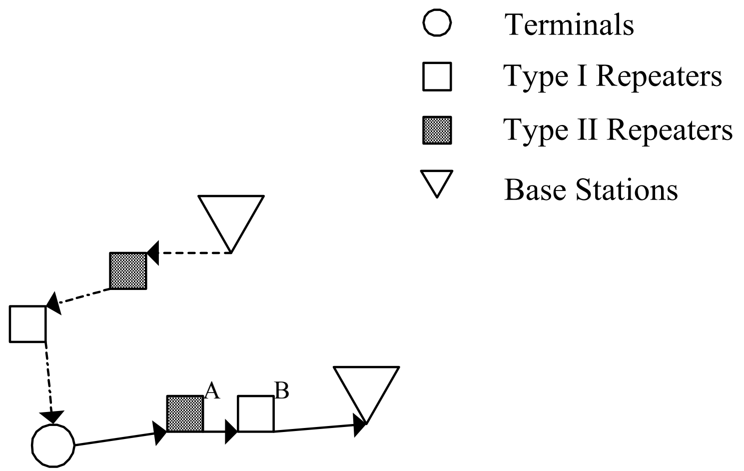
- Transmission Strategies
  - What should be the transmission range?
  - To whom should the transmission be addressed?
- Two methods have been investigated in the literature
  - NFP (Nearest with forward progress)
  - FFP (Farthest with forward progress)




NFP FFP

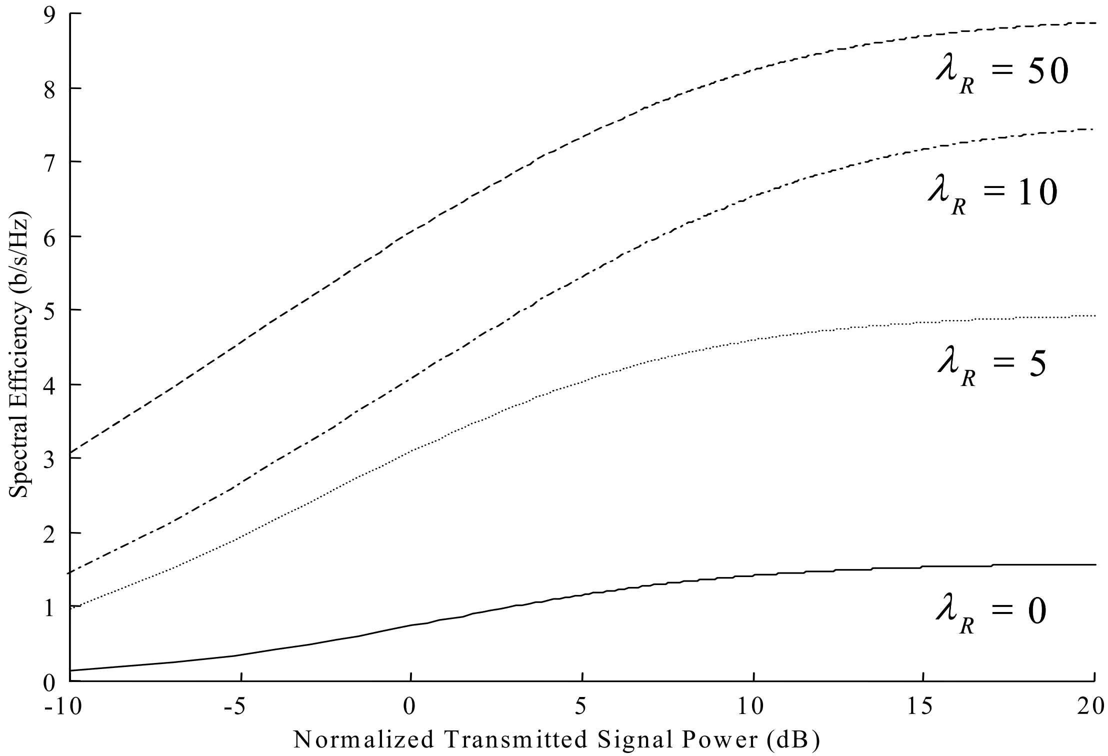
# Impact of Routing on Capacity (Cont'd)

- In the cellular structure the routing strategy is more complex
  - In uplink, each base station can be the final destination
  - In downlink, several base stations can send the data to one terminal
  - Loop free
  - Unilateral links



# A Typical System Performance

- System capacity for different routers density for network with multiuser detection and diversity techniques



# Performance Analysis

- Throughput ($N_s$) : The number of successfully received packets at a receiver at each slot
- $N_s$ is a binomial random variable with parameters :
  - $p_s$ : Probability of a packet successful reception
  - $N$ : Number of packets transmitted to that receiver
- $p_s$ is a function of $N$ and Interference due to transmissions to other receivers, $I$

$$
\begin{aligned}
\gamma &= E(N_s) = E(E(N_s|N, I)) \\
&= E(Np_s(N, I)) \\
&= \sum_{n=1}^{\infty} nE(p_s(n, I)) \Pr(N = n)
\end{aligned}
$$

# Performance Analysis (Cont'd)

- For the best very long codes

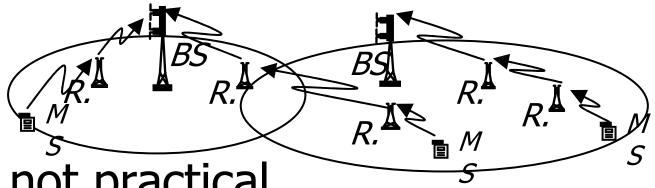
$$p_s(N,I) = \begin{cases} 1 & N + \frac{I}{P_x} < K(\mu_c) \\ 0 & N + \frac{I}{P_x} > K(\mu_c) \end{cases}$$

- Throughput

$$\gamma = \sum_{n=1}^{\lfloor K(\mu_c) \rfloor} n \Pr(I/P_x < K(\mu_c) - n) \Pr(N = n)$$

# System Model and Assumptions

- Two frequency bands carry the information on the uplink and downlink independently (FDD Mode)

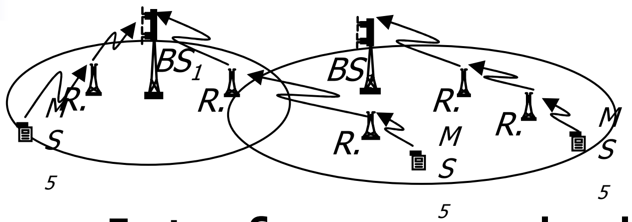



- In each frequency band
  - Simultaneous transmission and reception is not practical
  - Routers use TDD mode to send and receive data
  - Change their mode alternatively at the end of each slot
- Medium access technique
  - Code Division Multiple Access (CDMA)
  - All nodes may send and receive on one or several channels (spreading sequence) dynamically allocated by base stations
- Synchronization
  - All nodes are required to be synchronized to reduce packet collision
  - Exact synchronization is only necessary within the range of a few hops

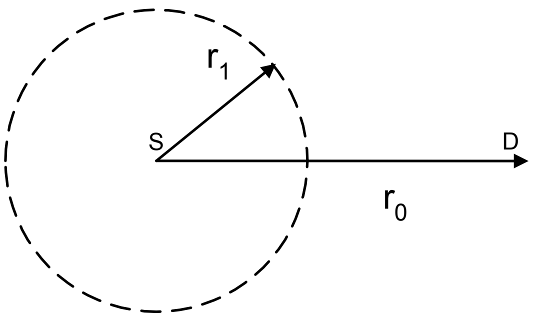
# System Model and Assumptions (Cont'd)

- Assumptions
  - Nodes are distributed as a two dimensional Poisson process
  - Channel access protocol is slotted ALOHA
  - In each slot, nodes try to transmit one packet with the fixed probability $p$ regardless of the number of received packets
  - In the uplink, ideal power control both at base station and repeaters

# Interference




- Interference calculation




$$f(r_0, r_1) = f(r_0) f(r_1 | r_0) = \frac{2r_0}{a^2} \begin{cases} 2\pi\lambda r_1 \exp(-\lambda\pi r_1^2) & r_1 < r_0 \\ \exp(-\lambda\pi r_0^2)\delta(r_0 - r_1) & r_1 = r_0 \\ 0 & r_1 > r_0 \end{cases}$$

$$u = r_1 / r_0$$

$$w = r_0$$

# Interference (Cont'd)

$$f_U(u) = 2(1 - e^{-Nu^2}(1 + Nu^2))/Nu^3 + (1 - e^{-N})\delta(u-1)/N$$

$$f_H(h) = \frac{1}{2}\frac{1 - e^{-N\sqrt{h}}(1 + N\sqrt{h})}{N\left(\sqrt{h}\right)^3} + \frac{1 - e^{-N}}{N}\delta(h-1)$$

$$\begin{aligned}\lim_{N\to\infty} N\int_0^1 h^n f_H(h)dh &= \lim_{N\to\infty}\int_0^1 \frac{1}{2}h^{n-3/2}(1 - e^{-N\sqrt{h}} - e^{-N\sqrt{h}}N\sqrt{h})\,dh + 1 \\ &= \int_0^1 \frac{1}{2}h^{n-3/2}dh + 1 = \frac{1}{2n-1} + 1 \qquad \text{for } n > 1/2\end{aligned}$$

# Interference (Cont'd)

$$M(s) = E(e^{sH}) = E\left(\sum_{k=0}^{\infty}\frac{(sH)^k}{k!}\right) = \sum_{k=0}^{\infty}\frac{s^k}{k!}E(H^k)$$

$$= 1 + \frac{1}{N}\sum_{k=1}^{\infty}\frac{s^k}{k!}(1+\frac{1}{(2k-1)}) + O(\frac{1}{N^2}) = \frac{N - i\sqrt{s}\sqrt{\pi}\,\mathrm{erf}\,(i\sqrt{s})}{N} + O(\frac{1}{N^2})$$

$$M_T(\omega) = E(e^{i\omega\sum H}) = \left(E(e^{i\omega H})\right)^N$$

$$= \lim_{N\to\infty}\left(1 - \frac{i}{N}\sqrt{(i\pi\omega)}\,\mathrm{erf}\left(i\sqrt{(i\omega)}\right) + O(\frac{1}{N^2})\right)^N$$

$$= \exp\left((1-i)\sqrt{\pi\omega/2}\,\mathrm{erf}\left(-(1-i)\sqrt{\omega/2}\right)\right) \quad \omega > 0$$

$$M_T(\omega) = M_T^*(-\omega)$$

$$f_{H_T}(h) = 2\,\mathrm{Re}\left\{\int_0^{\infty}\exp\left(-i\omega h + (1-i)\sqrt{\pi\omega/2}\,\mathrm{erf}\left(-(1-i)\sqrt{\omega/2}\right)\right)d\omega\right\}$$

# Conclusion

- Throughput calculation gives us a measure that can be easily compared with maximum system capacity
- Models should be developed for calculating throughput for networks which use different techniques for increasing capacity
- Routing algorithms proposed so forth for multihop networking are mostly based on efficient connectivity, further work need to be done to consider the effect of air interference

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