Analysis and Modeling of Upstream Throughput in Multihop Packet CDMA Cellular Networks

Ali Nabi Zadeh, *Member, IEEE*, and Bijan Jabbari, *Senior Member, IEEE*

***Abstract*—We consider the problem of throughput modeling of wireless multihop packet CDMA networks with cellular overlay using simple forwarding strategies in the upstream. Considering the effect of shadowing and distance-dependent path loss, we approximate the probability density of interference at each base station (BS) and compare numerical and simulation results for different path-loss parameters. We derive the probability density of the received power at each BS due to transmission of one packet from a random node, as well as the probability distribution of the number of packets received at each node per time slot. Subsequently, we use the above results to approximate the probability density of the total received power at each BS based on calculations of moments. We observe that the probability density of intercell interference due to transmissions from terminals and routers may be approximated by normal and log-normal densities, respectively. We quantify the network performance based on throughput, total consumed power, and outage probability for different system parameters. For homogeneous link efficiencies, introducing routers into the network while reducing the transmission power increases the mean and variance of interference to the desired signal, hence higher outage probability. However, there are ample opportunities inherent to multihop structure, applicable to any of the physical, data link, and network layers, which help increase the overall achievable network throughput.**

***Index Terms*—Ad hoc networks, multihop networks, network capacity, network modeling, wireless packet code-division multiple access.**

I. Introduction

THE capabilities of mobile cellular communications may be augmented by overlaying with a multihop network architecture [1], [2]. The primary motivation for considering this structure is to replicate the main attributes of the cell-splitting technique, namely, channel increase in space and reduction in power loss. By scaling down the cell size and the total number of channels in space, the network capacity can be linearly increased, proportional to the number of new base stations (BSs) or the scaling factor. Furthermore, shortening the links reduces the required transmit power. Although the power scaling does not increase the maximum network throughput in interference-limited networks, it does provide the opportunity for capacity increase when suitable techniques are deployed.

The problem with cell splitting is that small cells are often undesirable, as BSs and their interconnections to the wired backbone are typically costly. This problem can be overcome by deploying wireless routers, possibly placed randomly, in lieu of new BSs to establish a multihop wireless cellular network. Fig. 1 depicts a network structure comprised of BSs, routers, and terminals. In reality, selected mobile terminals may also act as routers, provided that they possess the necessary routing and relaying functionalities. Here, for clarity and without loss of generality, we assume that the terminal and router functions are assigned separately to individual entities in the radio network, and hence, a router will neither originate nor terminate traffic.

In this multihop wireless cellular network, traffic concentration, both in the upstream and downstream, is higher near the BSs due to packet relaying by intermediate nodes. Much the same as in cell splitting, shorter distances in the multihop structure result in a higher number of isolated links in space. However, since cell splitting only provides a scaled version of the original structure, capacity of an individual BS is not affected. Therefore, in spite of achieving a higher number of links when relaying is used, BSs still remain the bottleneck for system throughput. To improve performance, additional processing of information on the network is required in order to handle the excess interference and to trade off the conserved power for capacity advantage. In [2], we have addressed this problem and investigated through simulation the effect of deploying different techniques in the physical, data link, and network layers on network performance. We have shown that a combination of these techniques can be used to translate the high power saving into a multifold capacity increase.

This paper attempts to realistically model system throughput and outage probability associated with this structure. For a review of the architecture, signaling, and the forwarding mechanisms in multihop packet networks, the reader is referred to [3], where issues surrounding self-organized networks has also been discussed. In our modeling effort, we consider a direct-sequence code-division multiple-access (DS-CDMA) system, and analyze network performance for two forwarding strategies. Our motivation is to gain a better understanding of the potential capacity increase for multihop systems. We expect this effort to also provide a performance benchmark for evaluating different strategies and techniques in the physical, data link, and network layers, and allow optimizing network parameters without the need for time-consuming simulations.

The earliest investigations on performance modeling of a multihop structure were based on time-division multiple-access

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A. N. Zadeh was with the Communications and Networking Laboratory, George Mason University, Fairfax, VA 22030-4444 USA. He is now with Isocore Corporation, Reston, VA 20191 USA (e-mail: azadeh@ieee.org).

B. Jabbari is with the Electrical and Computer Engineering Department, George Mason University, Fairfax, VA 22030-4444 USA (e-mail: bjabbari@gmu.edu).

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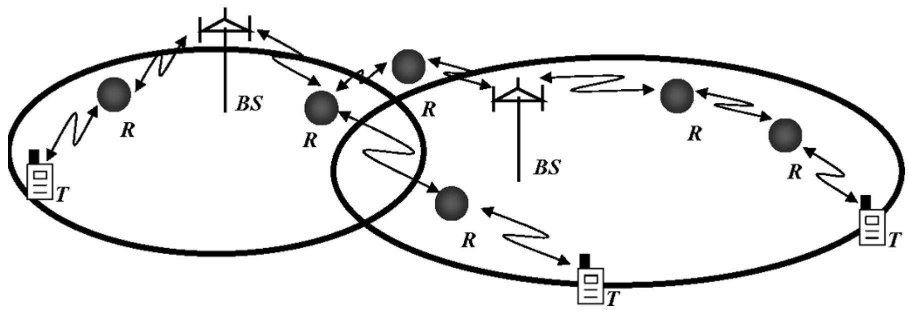

Fig. 1. Network structure: base stations (BS), routers (R), and terminals (T) are distributed uniformly in the network. Terminals in a cell might transmit to a BS in another cell based on the routing strategy.

(TDMA) pure ad hoc networks. The work reported in [4]–[6] consider a structure in which all nodes are of the same type and can relay the packets toward the destinations. The destination for the packets of each node is another randomly selected node. When the number of nodes sending packets to a relaying node is increased, the relaying node's probability of transmission and the number of packets it transmits in each slot may also increase. In these references, the probability of transmission and the number of packets are assumed to be constants, independent of network topology and forwarding strategies. Therefore, to account for actual traffic, expected forward progress (EFP) has been considered as the performance measure. EFP is defined based on the relationship between the total system traffic and the number of hops a packet traverses before it arrives at the destination. It is the expected value of the product of the distance a packet moves toward its destination in each hop and the probability of its successful reception per hop. It should be noted that although this measure is useful for comparing some forwarding strategies, it cannot predict the exact network throughput.

In [7], the same network configuration is used, and the work has been extended to DS-CDMA, investigating optimum transmission ranges. The paper assumes equal transmit power and equal transmission probabilities for all terminals. The propagation model is based on path loss, and shadowing has not been considered for calculating interference.

A considerable body of ongoing research is focused on the detailed analysis of new techniques for performance improvement in simpler architectures. Among recent work, [8] and [9] discuss dynamic power allocation and routing strategies for a pure ad hoc network considering time-varying channel states. Papers [10] and [11] discuss the details of employing spatial diversity techniques in multihop networks. The two diversity techniques which are due to the availability of multiroutes to destination and availability of multiple copies of packets due to multiple transmissions are specific to the multihop structure, and may also be deployed in the overlay structure. Time-allocation strategies have been investigated in [12] and [13]. All of the work above appears to be complementary to our work.

In this paper, our focus is on developing a model to evaluate upstream throughput, outage probability, and power consumption in a multihop network with cellular overlay where routers relay the packets until they arrive at any of the BSs. The novelty of this work lies in considering the exact traffic of routers and using network throughput as a performance measure, instead of a related measure, such as EFP.

The paper continues in Section II by describing the network structure, assumptions, and transmission scenarios. We introduce two forwarding strategies for upstream transmissions and discuss the complexity and expected system performance for each of them. In Section III, we explain the method and approach for derivation of the signal-to-noise-and-interference ratio (SNIR) at the BSs. In Section IV, we calculate the probability density function (pdf) of the total received power at a BS and describe the effect of forwarding strategies and shadowing variance within the model. We derive the pdf of the received power at each BS due to the transmission of one packet from a random node, as well as the probability distribution function (PDF) of the number of packets received at each node per time slot. We then approximate the pdf of the total received power based on calculations of moments. In Section V, we investigate the accuracy of our model by comparing the analytical and simulation results. Also examined in this section is the effect of shadowing variance and varying node densities on network performance. Finally, Section VI provides some concluding remarks.

II. System Model and Assumptions

A. Network Structure

The network assumed in this paper is comprised of BSs, routers, and terminals. BSs are assumed to be in the center of hexagonal cells with density per unit area λ_{BS}, while routers and terminals are both distributed in the plane as a 2-D Poisson point process with parameters λ_R and λ_T, respectively. Terminals transmit packets with probability p_t independently from slot to slot. The upstream and downstream transmissions may involve multiple routers, and therefore, routers must be able to receive on all the channels on which they transmit.

Although it is possible to employ the same frequency for upstream and downstream, here we consider two frequencies to operate in time-division duplex (TDD) mode (see Fig. 2) independently in each direction. In each frequency, time is divided into fixed slots, equal to the packet transmission time, and in each time slot, asynchronous CDMA is used as the access technique. Inherent fading and the interference averaging effect makes CDMA an attractive access scheme to achieve a

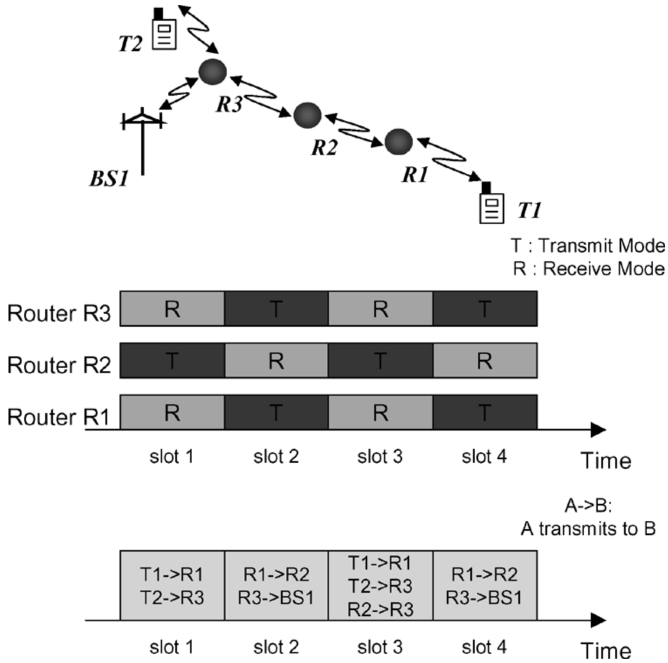

Fig. 2. Packet relaying in upstream or downstream, assuming terminals T1 and T2 always have packets to transmit. In each time slot, neighboring routers are assumed to be in different modes in order to be able to exchange data.

high system capacity and to implement simpler algorithms for dynamic channel assignment. Nodes may send and receive on several dynamically assigned spreading sequences. We assume that there are always a sufficient number of spreading codes available to be assigned to each node. BSs assign disjoint pools of spreading codes to neighboring routers so that they can avoid collision with other packet transmissions (for more information on spreading-code protocols, see [14]).

Nodes within the range of a few hops are required to be synchronized at the slot level in order to reduce packet collisions. The routers alternate modes at the end of each slot, hence, other nodes know when transmissions to them are possible. We assume that routers are in transmit or receive mode independent of their locations with a probability of one-half, and in each time slot, each router's mode is known to all its neighboring nodes in the network. Note that we do not consider any optimization on transmission-mode selection, and when a router needs to hand over to another router, it is responsible for searching within the set of routers with complementary transmission mode.[1]

B. Interference Model

In a wireless network, the radio channel is typically modeled as the product of three independent components: fading, shadowing, and distance loss. We assume that the fading component is fast fading, and is dealt with in the design of the encoder and performance analysis of the decoder as it is accounted for in the required symbol energy-to-noise-density ratio, $E_b/N_{0\text{eff}}$.

[1]Some recent studies have proposed scheduling algorithms to achieve higher achievable throughput in a variable slot duration and multirate network [15]; however, this is beyond the scope of this paper.

The effect of the shadowing and distance-loss components results in an attenuation that for a link length r can be expressed as

$$\alpha(r,\xi,\xi_x) = r^m 10^{\frac{q\xi}{10}} 10^{\frac{q_x \xi_x}{10}} \quad \text{for } m > 2, q^2 + q_x^2 = 1 \quad (1)$$

where ξ and ξ_x are two zero-mean normally distributed independent random variables with standard deviation σ describing the decibel attenuation due to shadowing, and m is the propagation exponent. We also assume that during the transmission of a data packet, channel attenuation $\alpha(r,\xi,\xi_x)$ is fixed and is known both to the transmitter and the receiver. Typical values for m and σ, based on experimental data, are 4 and 8 dB, respectively.

ξ_x models the propagation loss local to the node (near field), and so for a fixed source and different links, it has the same value. ξ models the propagation loss between the near field and the destination, and it is independent for links with different destinations. q can vary from zero, for the case of highly correlated attenuations, to one, for the case of independent attenuations in different directions. q and q_x are typically considered to be equal (for further discussion, refer to [16]). We also assume that the spatial variations of the shadowing loss are white, so that shadowing loss for any two distinguishable links are independent.

The noise at a detector is due to constant background thermal noise with power spectral density $N_0/2$, and interference is due to all transmissions in the same time slot and frequency band. We assume that the observed interference level at the detector is fixed during the reception of each packet. As will become clear in the next section, our model is independent of how packet success rate and interference level are related. However, to simplify the comparison of different scenarios, we consider the model developed in [17]. As shown in [17], for asynchronous DS-CDMA using binary phase-shift keying (BPSK) with a rectangular pulse shape, this interference can be modeled as Gaussian noise in the chip level. If the received packet from the desired channel has power P_0 and the total interference from other channels sums to I, then the effective symbol energy-to-noise-density ratio $E_s/N_{0\text{eff}}$ at the detector is equal to [17]

$$\mu = \frac{E_s}{N_{0\text{eff}}} = \left(\frac{2I}{3LP_0} + \frac{1}{\mu_0}\right)^{-1} \quad (2)$$

where L is the processing gain, and μ_0 is equal to E_s/N_0 at the detector.

The probability of packet success is dependent on the coding scheme. We denote the probability of packet success conditioned on μ as $p_s(\mu)$. In our model, there is no specific assumption on the rate-power curve; however, for simple comparison among different scenarios, we consider the use of optimum very long codes in the transceivers.

For the best very long codes, the function p_s approaches a step function at some value of μ, μ_c. Therefore, we can write p_s as

$$p_s(\mu) = \begin{cases} 1, & \mu : \frac{I}{P_0} < K(\mu_c) \\ 0, & \mu : \frac{I}{P_0} > K(\mu_c) \end{cases}$$
$$\text{where } K(\mu) = \frac{3L}{2}\left(\frac{1}{\mu} - \frac{1}{\mu_0}\right). \quad (3)$$

Again, for the best very long binary codes, the information rate of the code can approach the Shannon capacity limit [18]. Assuming the receivers benefit from high diversity order, the distribution of fading tends to follow a Gaussian distribution, and therefore, the information rate of packets R_P, and μ_c can be related by

$$R_P = \frac{1}{2}\log_2(1 + 2\mu_c) \text{ b/s/Hz.} \tag{4}$$

C. Packet Forwarding Strategy

We define the sample space S as the set of all possible configurations of the network and shadowing losses. We assume that system mobility and shadowing variations compared with packet time duration are quite slow, and thus the system can reach steady state for each realization of S. Therefore, a stochastic process can be defined by observing the traffic features for any of these outcomes in time. The routing strategies we are using determine the next hop based only on each outcome of S, and so the steady state is not a function of the initial values of the queue sizes of routers. Therefore, after a fixed time duration proportional to the average number of hops, these stochastic processes can be considered stationary processes. Since BSs are fixed and terminals and routers are uniformly distributed in the network, all outcomes in the sample space are equally probable.

We assume an inverse channel power control is employed for each packet transmission, and all the packets are received at their direct next hop with the same power level. As it is shown in [19], for mild fading conditions, the link capacity for all common adaptation techniques are within 3 dB of each other. Note that in this network architecture, due to the availability of multiroute diversity, the probability that all links are severely faded is diminished geometrically, and therefore, we can expect that the channel-inversion scheme will perform very closely to the optimal adaptation technique. However, this statement is only valid when interference and power-control strategies are completely independent. We defer this discussion of optimum power control to future work.

For each receiver, we use the terms intracell and intercell "interference" based on whether a transmission is destined to that receiver or not. In other words, to calculate the intracell interference at a receiver, we need to sum up the received power of all the packets destined to that receiver. Since these packets are all received with the fixed power P_0, the intracell interference is a discrete random variable equal to the number of packets destined at the receiver multiplied by P_0. The summation of the received power from all other transmissions will result in total intercell interference. Therefore, intercell interference is a continuous random variable.

Note that even if packets are generated independently from slot to slot, intercell and intracell interference are not independent. For example, consider a BS at the center of a cell. Received traffic at each router in the cell is dependent on the specific outcome of S, and therefore, is correlated in consecutive time slots. At the same time, the portion of transmitted packets which create the majority of intercell interference at a BS in one time slot contribute with a high probability to the intracell interference at that BS in the following time slot. Therefore, the intercell and intracell interference in the current slot are correlated. For this reason, to achieve higher accuracy in system modeling, we are using two different methods for calculations of intercell and intracell interference.

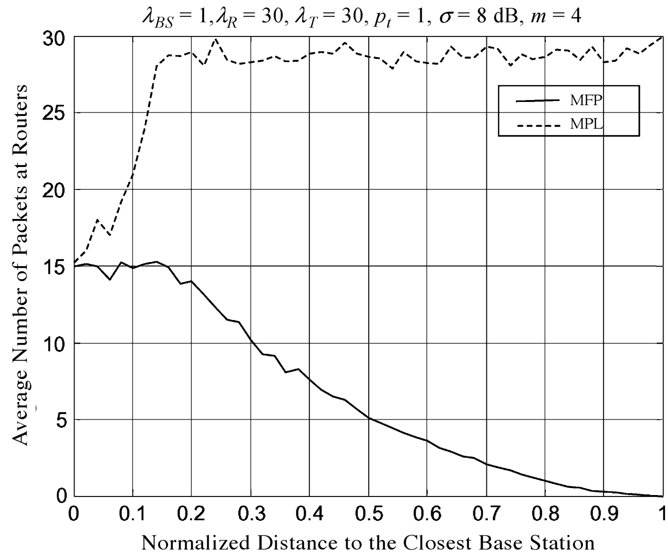

Fig. 3. Density of packets at receivers versus distance to the center of the cell.

We consider two simple strategies for forwarding packets toward the BSs and defer the discussion of optimum routing to future work. The two types of forwarding are as follows.

- Minimum Path Loss (MPL): In this strategy, a transmitter sends data on a link whose propagation loss is the minimum among the links to all possible next hops.
- MPL with Forward Progress (MFP): Consider the line which connects the transmitter to its closest BS. Transmissions to the nodes in the half-plane whose border is orthogonal to this line at the transmitter's location and includes the BS are considered to be transmissions with forward progress. We also refer to this half-plane as the forward progress region. In MFP, a transmitter searches within this half plane for the next hop, and sends its packets to the receiver for which the link propagation loss is minimum.

In our forwarding strategies, we assume that nodes will transmit only when their corresponding receivers are in receive mode. Fig. 3 shows the simulation results for the density of packets at receivers as a function of distance from the center of the cell. For a low probability of error, the average density of packets at a BS is the same as the average generated packets per cell.

Now assume that there is a router located very close to a BS. In these two forwarding strategies, there is no priority on selecting a BS over a router, except that routers cannot be used when they are part of a loop. Therefore, both in MFP and MPL, when the router close by the BS is in receive mode, on average, it will receive half of the total traffic directed toward the BS.

In MFP, the density of packets at a router decreases as its distance from the center of the cell increases until it is zero at the edge of the cell. In MPL, on the other hand, due to backward relaying, the density of packets at a router increases to the value at the BS as its distance from the center of the cell increases. For a large number of routers, we can assume that the average density of packets at routers at different locations and at BSs are equal.

We also assume that packets maintain a history of the hops they have traversed, and therefore, by deleting the old routers from their possible next hop list, a loop-free routing strategy is obtained. Both in MPL and MFP, some of the routers may transmit on links with higher propagation loss to avoid the loops. This results in higher interference values and higher error rate at routers, as compared with BSs. In the worst case, there are routers which are completely blocked and cannot be used to forward packets toward the destinations.

Methods to avoid the blocking of routers by neighbors have been addressed in [20]. As shown in later sections, MFP has much better performance than MPL. It should be noted that practical routing scenarios will follow a similar packet-density pattern as in the MFP case. Therefore, assuming that blocking of routers has been sufficiently addressed, in practical routing scenarios, since the traffic directed toward routers is much less than the traffic directed toward BSs, the required multiuser detection capability at routers is less than at BSs. We will, therefore, focus on the calculation of the BSs' performance as they are the throughput bottleneck, and assume there is no error due to transmissions to routers.

The required path-loss information in the MPL case can be obtained by assigning a pilot signal with fixed transmit power to both routers and BSs. In MFP, location information is also required, which can be acquired by equipping all the nodes with a global positioning system (GPS) receiver. In practical scenarios, the routes are selected based on the aggregate cost for transmission to the final destination, and location information is not required [21]. However, the two forwarding strategies introduced in this paper are used to give a closed-form formulation for achievable throughput as a function of different network parameters. In MFP, since the direction of transmission is already known, the chance of creating loops is intrinsically much lower than MPL. Therefore, there are fewer assumptions in the performance calculations of MFP, and a more accurate result is expected.

III. Network Performance Model

In this section, we develop a performance model based on computing the probability distribution of the total received power at the BSs and its approximation. All the probability densities and distributions are calculated over the sample space S of all possible network topologies and shadowing losses. The performance measure of interest can be outage probability, achievable throughput, or power consumption. We divide the problem into different steps, and in each step, investigate the accuracy of our assumptions.

A. Total Received Power

The distribution of the total received power at a BS is calculated based on the pdf of the received power from each transmitted packet. However, we first discuss the dependency of the received power from different transmitted packets. There are three sources contributing to this received power dependency, as follows.

1) Transmission of packets from the same node: The transmission rate of routers must be proportional to the ratio of terminal density to router density in order to avoid congestion. This requires allowing routers to transmit more than one packet in each time slot. Similarly, when terminals are allowed to transmit more than one packet per time slot, each packet will cause the same level of interference at all receivers. The complete dependency is for those packets that are residing at the same node and are transmitted to the same local destination with equal transmit power.
2) Dependency of the received power to the network topology: The received power due to the transmission of each packet and the network structure are correlated. In other words, as the received power levels due to the transmissions of more packets are known to the receiver, more information about the network structure is available. It is also clear that the pdf of the received power from each node is a function of its location statistics, and therefore, knowledge of the received power from one node affects the pdf of the received power from another node. Heuristically, this is also a function of the forwarding strategy. However, this correlation must not be very high, particularly when shadowing is present, and the forwarding strategy is based on the path loss and not the distance loss. This dependency becomes weaker as the variance of the shadowing increases. For a very high shadowing variance, the next hop for a packet is not dependent on the network topology.
3) Correlation in the traffic of neighboring routers: As explained above, the received traffic at a router is proportional to the number of nodes that have that router as their next hop, and therefore, the traffic arrivals at neighboring routers are correlated.

However, throughout this paper, we only consider the first source of dependency, and investigate the effect of the other sources on the accuracy of the analysis by comparing with the simulation results. Our method models the system performance at independent time slots. Since our basic assumption is independency of the received power due to transmissions of different nodes, the result will be more accurate for a Bernoulli traffic source with no time correlation. However, if the traffic characteristics for all the sources are identical, we can still find the probability that each node in each slot has a packet to transmit, and use this model to predict the performance.

If the density of routers is much larger than the density of BSs, the total received power at any BS is mainly due to packet transmissions from nodes in the same cell. Therefore, knowing the distribution of the number of packets at each node, for calculation of the received power, it is sufficient to assume that the network is comprised of one BS with terminals and routers distributed uniformly around it in the infinite plane.

Consider a receiver centered in a circular region R_a with radius a, and let H_i be the received power at this receiver from the ith transmitter located in this region. Therefore, $H_T = \sum_i H_i$, with the summation over all transmitters in the region R_a, denotes the total received power, interference plus desired signal power, at this node when $a \rightarrow \infty$. We derive the moment generating function (MGF) for H_T as follows:

$$\begin{aligned} E(e^{sH_T}) &= E\left(e^{s\sum_i H_i}\right) \\ &= \sum_{k=0}^{\infty} e^{-\pi a^2 \lambda_{\mathrm{Tx}}} \frac{(\pi a^2 \lambda_{\mathrm{Tx}})^k}{k!} \\ &\quad \times E\left(e^{s\sum_i H_i} | k \text{ in } R_a\right) \end{aligned} \tag{5}$$

where "k in R_a" is the event that there are k transmitting nodes in region R_a, and λ_{Tx} is the average number of transmitting nodes per unit area. For H_i's with identical distribution (routers only or terminals only), as we have assumed that all H_i's are independent of each other, we can write

$$\begin{aligned} E(e^{sH_T}) &= \sum_{k=0}^{\infty} e^{-\pi a^2 \lambda_{\mathrm{Tx}}} \frac{(\pi a^2 \lambda_{\mathrm{Tx}})^k}{k!} \left[E(e^{sH_1})\right]^k \\ &= \exp\left(\pi a^2 \lambda_{\mathrm{Tx}} \left(E(e^{sH_1}) - 1\right)\right). \end{aligned} \tag{6}$$

We calculate the characteristic function of the received power from routers and terminals separately, and multiply them to obtain the characteristic function of the total received power. The pdf of the total received power can be obtained by inverting the characteristic function and using the inverse fast Fourier transform to calculate it numerically. In the next subsection, we look at an alternative method for a direct estimation of the pdf.

B. Approximation for Received Power Distribution

We use the Charlier series [22] as a general method to approximate the pdf of the intercell interference based on the knowledge of its cumulants and the derivatives of a known distribution. Let $\psi(t)$ be the characteristic function of the function $\Psi(t)$, and γ_r its cumulants. Similarly, let $F(t)$ be the distribution to be approximated, $f(t)$ its characteristic function, and κ_r its cumulants. By definition, these quantities are connected by the formal series

$$f(t) = \exp\left(\sum_{r=1}^{\infty} \frac{(\kappa_r - \gamma_r)(it)^r}{r!}\right) \psi(t). \tag{7}$$

Integrating by parts gives $(it)^r \psi(t)$ as the characteristic function of $(-1)^r \Psi^r(x)$, so the formal identity corresponds pairwise to the identity

$$F(x) = \exp\left(\sum_{r=1}^{\infty} \frac{(\kappa_r - \gamma_r)(-D)^r}{r!}\right) \Psi(x) \tag{8}$$

where D is the differential operator.

Here, we expand the pdfs of the received power levels from terminals and routers based on normal and log-normal distributions. Now based on the Charlier expansion, to calculate the pdf of the total received power H_T, we only need to obtain the first few cumulants of H_T. Let $(\kappa_r)_{r=1}^{\infty}$ denote the rth cumulants of H_T, therefore

$$\begin{aligned} \kappa_r &= \frac{d^r}{ds^r} \left(\ln E(e^{sH_T})\right)|_{s=0} \\ &= \frac{d^r}{ds^r} \left(\pi a^2 \lambda_{\mathrm{Tx}} E(e^{sH_1})\right)|_{s=0} \\ &= N_{\mathrm{Tx}} E\left(H_1^r\right) \end{aligned} \tag{9}$$

in which $N_{\mathrm{Tx}} = \pi a^2 \lambda_{\mathrm{Tx}}$.

As a result, the cumulants of H_T can be calculated based on the moments of the received power from one node. Letting a approach infinity, to calculate the cumulants of the total received power, we only need to calculate the moments of H_1 up to the order of $1/a^2$. Note that the first and second cumulants are, respectively, equal to the mean and variance.

C. Performance Measure

The number of successfully received packets per time slot at a receiver, N_s, is a binomial random variable with parameters p_s, the probability of packet success, and N, the number of packets transmitted to that receiver. This is because due to the inverse channel power-control strategy, all these packets undergo the same level of noise plus interference and are received with the same power level. p_s is a function of random variables N and H_T, the total received power due to all transmissions. Then the per-BS throughput γ, in terms of b/s/Hz, is given by

$$\frac{\gamma L}{R_p} = E(N_s) = E\left(E(N_s|N, H_T)\right) = E\left(N p_s(N, H_T)\right) \tag{10}$$

where the inner expectation is over all different codewords, while the outer expectation is over different outcomes in the sample space S when in steady state. R_p and L are the information rate per packet and processing gain, respectively.

Based on the assumption that received power levels at a receiver due to the transmissions from different nodes are independent, (10) can be simplified as

$$\frac{\gamma L}{R_p} = \sum_{n=1}^{\infty} n E\left(p_s(n, H_T)|N = n\right) \Pr(N = n). \tag{11}$$

Again, for the best very long codes, we can further simplify this formula by substituting (3) into (11) and obtaining

$$\frac{\gamma L}{R_p} = \sum_{n=1}^{\lceil K(\mu_c) \rceil + 1} n \Pr\left[\frac{I_{\mathrm{intercell}}}{P_0} < K(\mu_c) + 1 - n\right] \Pr\left[N = n\right] \tag{12}$$

where $I_{\mathrm{intercell}}$ is the intercell interference, and P_0 is the fixed required received power at the BS. We have also replaced H_T by $NP_0 + I_{\mathrm{intercell}}$. Since packets are received with the same power level, outage probability can be defined as the percentage of slots where the BS cannot successfully decode any packet

$$P_{\mathrm{out}} = \Pr\left[\frac{(H_T - P_0)}{P_0} > K(\mu_c)\right]. \tag{13}$$

Assume a node is transmitting Λ_p packets in a time slot on a link with attenuation $\alpha(r,\xi,\xi_x)$. Since the required received power is a fixed value P_0, the average consumed power per node can be obtained as

$$P_T = E\left[P_0\Lambda_p\alpha(r,\xi,\xi_x)\right] = e^{\frac{(q_x(\ln 10)/10)^2\sigma^2}{2}} E\left[P_0\Lambda_p r^m 10^{\frac{q\xi}{10}}\right]. \quad (14)$$

IV. Probability Density Function of Interference

All the performance measures discussed in this paper can be calculated by knowing the pdf of the total interference. In this section, for each forwarding strategy, we first calculate the intercell interference pdf and the intracell interference PDF separately, and then use these functions to estimate the pdf of the total observed interference at a BS. Note that the procedures for calculating the received power from terminals and routers are the same, except that the density of packets at terminals and routers is different.

A. List of Parameters Used in Analysis

P_0	The fixed power level which is required for each packet to be received at its immediate next-hop decoder.
λ_T	Density of terminals per unit area.
λ_R	Density of routers per unit area.
Λ_p	A random variable describing the number of packets to be transmitted in a time slot from a node. For terminals, this is equal to p_t, and for routers, it should be obtained based on the deployed forwarding strategy.
N	A random variable which denotes the number of packets transmitted to a receiver in a time slot. For low probability of packet loss, N at a time slot t and Λ_p at the time slot $t+1$ are equal for routers.
λ_{Tx}	Density of the transmitting nodes per unit area. To obtain the pdf of intercell interference due to transmissions from terminals and routers, this will be set to λ_T and $\lambda_R/2$, respectively.
λ_{Rx}	Density of routers in receive mode per unit area. Due to random mode selection, it is equal to $\lambda_R/2$.
p_t	Transmit probability of each terminal per time slot.
H_i	Received power at the BS in the center of region R_a.
H_T	Total received power at the BS in the center of region R_a.

B. MPL With Fixed Received Power—Intercell Interference

Assume λ_{Tx} is the density of the transmitting nodes per unit area. Our model for this forwarding strategy is based on the following two assumptions whose validity were discussed in Section III-A:

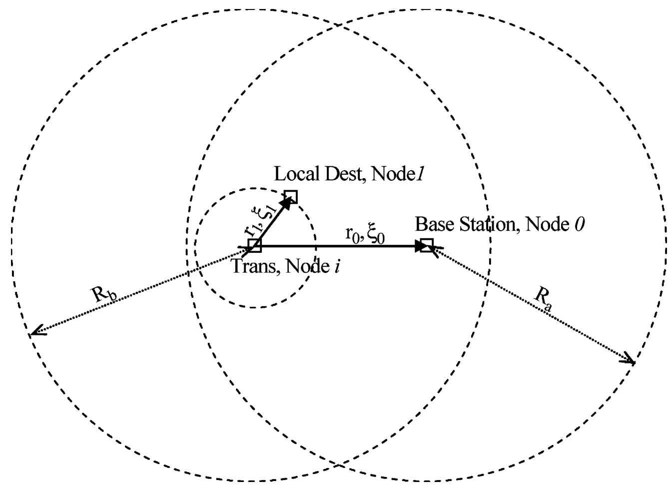

Fig. 4. Interference due to a source which is located at a distance r_0 from the center, and transmits to a local host at distance r_1. ξ_0 and ξ_1 are the far-field decibel attenuation due to shadowing on these two links.

- the statistics of the number of packets that each node transmits in a time slot is independent of the node's distance from the center of its cell site;
- the interference observed at a receiver due to the transmission of one packet from a node is independent of the number of packets buffered at that node.

Once again, consider the circular region R_a with a BS, identified as node 0, located at its center. Let r_0 be the distance of a node i from the center of the region, and r_1 be the distance of this node from its immediate next hop, identified as node 1 (see Fig. 4). Also, let ξ_0 and ξ_1 denote the far-field decibel attenuation due to shadowing on these two links. Note that due to the inverse channel power-control strategy, near-field shadowing attenuation does not affect the calculation of interference, and therefore, for simplicity, we define the link attenuation on a link j as $\alpha(r_j,\xi_j) = r_j^m 10^{q\xi_j/10}$.

Let us denote by Λ_p the number of packets at this node's buffer in each time slot, and by P_0 the required received power for each packet at its designated receiver. Note that P_0 is the same for all receivers in the network. Therefore, if we normalize all the power levels to P_0, the received power H_i from node i at the center of the cell can be written as $H_i = \Lambda_p(r_1/r_0)^m 10^{q(\xi_1-\xi_0)/10}$.

By introducing the new random variable $U = (r_1/r_0) 10^{q(\xi_1-\xi_0)/10m}$, the received power H_i can be simplified as $H_i = \Lambda_p U^m$. Now, using the independency assumption, the cumulants of H_T can be calculated based on the moments of U and Λ_p. In this section, we obtain the pdf of U and leave the calculation of the PDF of Λ_p to the section on intracell interference calculation.

To calculate the pdf of U, we start by obtaining the joint pdf of (r_0,ξ_0,r_1,ξ_1). Since far-field components of shadowing are independent of each other and of locations of receivers, the joint pdf of (r_0,ξ_0,r_1,ξ_1) can be expanded as follows:

$$f(r_0,\xi_0,r_1,\xi_1) = \left(\frac{2r_0}{a^2}\right) f(\xi_0) f(\xi_1) f(r_1|r_0,\xi_0,\xi_1) \quad (15)$$

where the first term is the probability that node i is located at distance r_0 from the BS.

Consider the transmitter node i in Fig. 4 to be located in the center of a circular region R_b with radius b. Let A_j, $j = 0$ or 1, be the event that conditioned on (r_0, ξ_0, ξ_1), the transmitter selects node j, located at distance r_j from it in region R_b as its next hop. In other words, the propagation loss from the transmitter to receiver j is minimum among all other links. Based on our assumption for routing strategy, $f(r_1|r_0, \xi_0, \xi_1)$ can be written as

$$\begin{aligned} f(r_1|r_0, \xi_0, \xi_1) = \lim_{b\to\infty} & 2\pi\lambda_{\mathrm{Rx}} r_1 \Pr[A_1] I \\ & \times (\alpha(r_1, \xi_1) < \alpha(r_0, \xi_0)) \\ & + \Pr[A_0]\delta(r_1 - r_0) I(\xi_1 = \xi_0) \end{aligned} \quad (16)$$

in which λ_{Rx} is the density of receivers, $\delta(\cdot)$ is the Dirac delta function, and $I(\cdot)$ is the indicator function which equals 1 only when its argument is true, and otherwise equals 0. Then (see [20] and [23])

$$\Pr[A_j] = \exp\left(\pi b^2 \lambda_{\mathrm{Rx}} \left(\Pr\left[\alpha(r_j, \xi_j) \le \alpha(r_l, \xi_l)\right] - 1\right)\right) \quad (17)$$

where (r_l, ξ_l) denotes the link parameters between a random node in R_b and the source. This formula can further be simplified as

$$\begin{aligned} &\Pr\left[\alpha(r_j, \xi_j) \le \alpha(r_l, \xi_l)\right] \\ &\qquad = \int_0^b Q\left(\frac{\left(\xi_j - \frac{m \ln\left(\frac{r_l}{r_j}\right)}{\beta}\right)}{\sigma}\right) \frac{2r_l}{b^2} dr_l \end{aligned} \quad (18)$$

where $\beta = q \ln 10/10$ and $Q(x)$ is the standard Q-function $Q(x) = \int_x^\infty (1/\sqrt{2\pi}) e^{-t^2/2} dt$.

Defining $t = (\xi_j - m \ln(r_l/r_j)/\beta)/\sigma$ and integrating by parts, we can write

$$\begin{aligned} &\lim_{b\to\infty} b^2 \left(\Pr\left[\alpha(r_j, \xi_j) \le \alpha(r_l, \xi_l)\right] - 1\right) \\ &= \lim_{b\to\infty} \left(Q(t) r_l^2\Big|_0^b - b^2\right) - \int_0^b r_l^2 \frac{dQ(t)}{dr_l} dr_l \\ &= \lim_{b\to\infty} b^2 Q\left(\frac{\left(-\xi_j + \frac{m \ln\left(\frac{b}{r_j}\right)}{\beta}\right)}{\sigma}\right) + \int_{+\infty}^{-\infty} \frac{r_l^2}{\sqrt{2\pi}} e^{-\frac{t^2}{2}} dt \\ &= -r_j^2 e^{\frac{2\beta\xi_j}{m}} \int_{-\infty}^{+\infty} \frac{1}{\sqrt{2\pi}} e^{-\frac{2\beta\sigma t}{m}} e^{-\frac{t^2}{2}} dt \\ &= -r_j^2 e^{\frac{2\beta\xi_j}{m}} e^{\frac{2\beta^2\sigma^2}{m^2}}. \end{aligned} \quad (19)$$

The first term in the limit has been removed based on the inequality $Q(x) \le (1/2) e^{-x^2/2}$ for $x > 0$.

Finally, the pdf $f(r_1|r_0, \xi_0, \xi_1)$ is obtained as

$$\begin{aligned} f(r_1|r_0, \xi_0, \xi_1) = \exp&\left(-\pi\lambda_{\mathrm{Rx}} r_1^2 e^{\frac{2\beta\xi_1}{m}} e^{\frac{2\beta^2\sigma^2}{m^2}}\right) \\ &\times \left(2\pi\lambda_{\mathrm{Rx}} r_1 I\left(\alpha(r_1, \xi_1) < \alpha(r_0, \xi_0)\right)\right. \\ &\left. + \delta(r_1 - r_0) I(\xi_1 = \xi_0)\right). \end{aligned} \quad (20)$$

Now, substituting u defined as $u = (r_1/r_0)\exp(\beta(\xi_1 - \xi_0)/m)$ in the above formula, we can obtain

$$\begin{aligned} &f(u|r_0, \xi_0, \xi_1) \\ &\quad = \exp\left(-\pi\lambda_{\mathrm{Rx}} r_0^2 u^2 e^{\frac{2\beta\xi_0}{m}} e^{\frac{2\beta^2\sigma^2}{m^2}}\right) \\ &\quad\quad \times \left(r_0 \exp\left(-\frac{\beta(\xi_1 - \xi_0)}{m}\right) \times 2\pi\lambda_{\mathrm{Rx}} u r_0 \right. \\ &\quad\quad\quad \left. \times e^{-\beta\frac{\xi_1 - \xi_0}{m}} I(u < 1) + \delta(u - 1)\right). \end{aligned} \quad (21)$$

Note that the Jacobian term does not change the $\delta(u-1)$ term. Now, by integrating out r_0 from $f(u, r_0|\xi_0, \xi_1)$, the distribution for u is obtained as

$$\begin{aligned} f_U(u|\xi_0, \xi_1) = & \frac{2}{N_{\mathrm{Rx}} u^3} \exp\left(-2\beta \frac{m\xi_0 + 2\beta\sigma^2 + m\xi_1}{m^2}\right) \\ & \times I(u < 1) + \frac{1}{N_{\mathrm{Rx}}} \exp\left(-2\beta \frac{m\xi_0 + \beta\sigma^2}{m^2}\right) \\ & \times \delta(u - 1) + o\left(\frac{1}{a^2}\right) \end{aligned} \quad (22)$$

in which $N_{\mathrm{Rx}} = \lambda_{\mathrm{Rx}} \pi a^2$ and $o(k(a))$ is equivalent to $\lim_{a\to\infty}(o(k(a))/k(a)) \to 0$.

It is easy to show that for calculation of the moments of interference, the second terms in the above formulas do not affect the final results. Therefore, hereafter we remove these terms while keeping in mind that without them, the integral of f will not equal 1. Integrating out ξ_0, ξ_1, we see that interestingly, the result is not a function of shadowing variance. However, as discussed earlier, higher shadowing variance results in a more accurate model due to reducing the dependency between the received power from different nodes. Therefore

$$f_U(u) \approx \frac{\left(\frac{2}{u^3} + \delta(u - 1)\right)}{N_{\mathrm{Rx}}}. \quad (23)$$

The first term in the above formula contributes to the total intercell interference, and the second term to intracell interference. The cumulants of H_T in the limit as $a \to \infty$ are obtained as

$$\begin{aligned} \kappa_k &= N_{\mathrm{Tx}} E\left(H_1^k\right) \\ &= N_{\mathrm{Tx}} E\left(\Lambda_p^k U^{mk}\right) = N_{\mathrm{Tx}} E\left(\Lambda_p^k\right) E(U^{mk}) \\ &= E\left(\Lambda_p^k\right) \lim_{a\to\infty} N_{\mathrm{Tx}} \int_0^1 u^{mk} f_U(u) du \\ &= E\left(\Lambda_p^k\right) \frac{\lambda_{\mathrm{Tx}}}{\lambda_{\mathrm{Rx}}} \left(1 + \frac{1}{\frac{mk}{2} - 1}\right) \quad \text{for } k \ge 1 \text{ and } m > 2. \end{aligned} \quad (24)$$

Note that the average of the total interference is linearly dependent on $\lambda_{\mathrm{Tx}}/\lambda_{\mathrm{Rx}}$.

C. MFP With Fixed Received Power—Intercell Interference

For the MFP case, the difference from MPL is that the next hop candidates for each transmitter are the nodes in the forward progress region. Thus, the solution is a simple extension of the case of MPL with fixed received power. Therefore, we only address the steps that are different from the MPL case. We assume that the density of packets at a router is only a function of the distance of the router from the center of the cell. Therefore, the received power H_i from the source in Fig. 4 can be written as $H_i = \Lambda_p(r_0)U^m$.

Consider the source node in Fig. 4 to be located in the center of a circular region R_b with radius b. The search in the half-plane modifies the calculation of $f(r_1|r_0,\xi_0,\xi_1)$ as follows:

$$\begin{aligned}
&f(r_1|r_0,\xi_0,\xi_1)\\
&\quad=\lim_{b\to\infty}\pi\lambda_{\mathrm{Rx}}r_1\Pr[A_1]I\left(\alpha(r_1,\xi_1)<\alpha(r_0,\xi_0)\right)\\
&\qquad+\Pr[A_0]\delta(r_1-r_0)I(\xi_1=\xi_0)\\
&\Pr[A_j]\\
&\quad=\sum_{k=0}^{\infty}e^{-\frac{\pi b^2\lambda_{\mathrm{Rx}}}{2}}\frac{\left(\frac{\pi b^2\lambda_{\mathrm{Rx}}}{2}\right)^k}{k!}\Pr\left[A_j|k\text{ in }R_b^+\right]\\
&\quad=\exp\left(\frac{\pi b^2}{2}\lambda_{\mathrm{Rx}}\left(\Pr\left[\alpha(r_j,\xi_j)\le\alpha(r_l,\xi_l)\right]-1\right)\right)
\end{aligned}\tag{25}$$

where "k in R_b^+" is the event that there are k other transmitting nodes in the forward progress region R_b^+, and (r_l,ξ_l) denotes the link parameters between each of these nodes and the source. Following the same steps as in the MPL case, calculation of $f(u|r_0,\xi_0,\xi_1)$ will result in

$$\begin{aligned}
f(u|r_0,\xi_0,\xi_1)=\exp&\left(-\frac{\pi}{2}\lambda_{\mathrm{Rx}}r_0^2u^2e^{\frac{2\beta\xi_0}{m}}e^{\frac{2\beta^2\sigma^2}{m^2}}\right)\\
\times&\left(\pi\lambda_{\mathrm{Rx}}ur_0^2\exp\left(-\frac{2\beta\left(\xi_1-\xi_0\right)}{m}\right)\right.\\
&\left.\times I(u<1)+\delta(u-1)\right).
\end{aligned}\tag{26}$$

For calculation of the moments of H_i in the limit as $a\to\infty$, the difference from the MPL case is that the density of packets at each router is a function of distance r_0

$$\begin{aligned}
\kappa_k&=N_{\mathrm{Tx}}E\left(H_1^k\right)=N_{\mathrm{Tx}}E\left(E\left(\Lambda_p^k(r_0)U^{mk}|r_0\right)\right)\\
&=N_{\mathrm{Tx}}E\left(\lambda_{p,k}(r_0)E(U^{mk}|r_0)\right)\\
&=N_{\mathrm{Tx}}E\left(\lambda_{p,k}(r_0)U^{mk}\right)
\end{aligned}\tag{27}$$

where $\lambda_{p,k}(r_0)$ is the kth moment of the density of packets at the source when it is located at distance r_0. Therefore, for moments calculation, the modification of the MPL case is only to weight $f_U(u|r_0,\xi_0,\xi_1)$ by $\lambda_{p,k}(r_0)$ and integrate out r_0, ξ_0, and ξ_1. We denote this new function by $g_U(\cdot)$

$$g_U(u|r_0,\xi_0,\xi_1)=\lambda_{p,k}(r_0)f_U(u|r_0,\xi_0,\xi_1).\tag{28}$$

A simple formulation results if we approximate $\Lambda_p(r_0)$ by $\Lambda_p\exp(-(\pi/2)\lambda_{\mathrm{Rx}}lr_0^2)$, in which we have assumed that the number of packets at different nodes follow the same pattern, and Λ_p is the random variable associated with the number of packets at the receiver located by the BS at the center. l is an adjustment parameter which is a function of the nodes densities and will be calculated in the next subsection. If transmitting nodes are terminals, l will be set to zero, as their incoming traffic is not a function of distance. Approximating each cell by a circular region with radius r_c, we have $\pi r_c^2=1/\lambda_{\mathrm{BS}}$. Then

$$\begin{aligned}
g_U(u|\xi_0)&=\int_{-\infty}^{+\infty}\int_0^a\Lambda_p^k(r_0)f_U(u|r_0,\xi_0,\xi_1)f(r_0)f(\xi_1)dr_0d\xi_1\\
&=\frac{4\Lambda_p^k}{N_{\mathrm{Rx}}}\frac{u\exp\left(2\beta\frac{m\xi_0+\beta\sigma^2}{m^2}\right)}{\left(u^2\exp\left(2\beta\frac{m\xi_0+\beta\sigma^2}{m^2}\right)+lk\right)^2}I(u<1)\\
&\quad+\frac{2\Lambda_p^k}{N_{\mathrm{Rx}}}\frac{1}{\exp\left(2\beta\frac{m\xi_0+\beta\sigma^2}{m^2}\right)+lk}\delta(u-1)\\
&\quad+o\left(\frac{1}{a^2}\right).
\end{aligned}\tag{29}$$

And finally, substituting this formula in (28) yields

$$\begin{aligned}
\kappa_k=E\left(\Lambda_p^k\right)\frac{\lambda_{\mathrm{Tx}}}{\lambda_{\mathrm{Rx}}}
&\times\left(\int_{-\infty}^{+\infty}\int_0^1\frac{4u\exp\left(2\beta\frac{m\xi_0+\beta\sigma^2}{m^2}\right)}{\left(u^2\exp\left(2\beta\frac{m\xi_0+\beta\sigma^2}{m^2}\right)+lk\right)^2}du\,d\xi_0\right.\\
&\quad\left.+\int_{-\infty}^{+\infty}\frac{2}{\exp\left(2\beta\frac{m\xi_0+\beta\sigma^2}{m^2}\right)+lk}d\xi_0\right).
\end{aligned}$$

This shows that unlike the MPL case, the interference is a function of shadowing and will decrease as shadowing variance increases, on condition that we are using perfect power control. Using this formula, the cumulants of the received power can easily be calculated numerically.

D. Density of Packets at Routers/BSs—Intracell Interference

So far, we have calculated the pdf of the total intercell interference based on the assumption that received power due to different nodes are independent. However, this assumption becomes weaker for the routers which are only one hop away from their cell's BS. For example, it is clear that as the number of these routers increases, the number of packets that each of these routers relays to the BS reduces. Therefore, we only use the independency assumption for intercell interference, which, as defined previously, is due to nodes being more than one hop away from the BSs. In this subsection, we complete the calculation of the total interference level by obtaining the density of packets destined to routers and BSs in each time slot.

Let us first consider the case of MFP. In this strategy, the final destination of the packets from each terminal is always the BS in the same cell. Therefore, for each BS, in each time slot, a Poisson number of packets are generated. With no buffering of packets at terminals and the fact that the next hop for each transmitting node is only determined based on the instance of link loss matrix, the generated packets from a terminal will traverse no more than two distinct paths before they reach their BS. Due to the independency of transmissions from slot to slot, in two consecutive time slots, the probability of receiving a packet at the head of the route is equal to the probability of forwarding one packet from the tail of the route. In other words, in any two consecutive time slots, each route forwards one packet to the BS with probability of p_t.

Thus, in the steady state and for two consecutive time slots, the total number of packets generated in one cell and the total number of packets delivered to the BS have the same probability distribution.

Also, note that if the path involves a router, there is no prior knowledge on which time slot the packet is delivered to the BS. Therefore, for all the transmissions that are forwarded by routers, the density of packets received at the BS in one time slot, denoted by Z, conditioned on the total number of received packets in two consecutive time slots, denoted by X, has a uniform distribution in the range of 0 and $X = x$. Thus

$$P_{Z|X}(z|x) = \frac{1}{x} I(0 \leq z \leq x). \tag{30}$$

Simulation results suggest that a simple accurate formula is obtained if we assume that the number of packets transmitted on terminal-BS links and the number of packets transmitted on router-BS links are independent. One can expect to obtain considerably more accurate results for a high density of routers, where the probability of terminals transmitting directly to the BSs is rather low.

Let N_T be the number of terminals in a cell that transmit to the BS through routers. If we denote by p_{BS} the probability that a terminal transmits directly to the BS in a cell, N_T has a Poisson distribution with mean $(1 - p_{\mathrm{BS}})\lambda_T/\lambda_{\mathrm{BS}}$. Due to limited node mobility in a slot time scale, the number of terminals in each cell in consecutive time slots remains almost constant. Therefore, the total number of packets received at a BS, through routers in two consecutive time slots, conditioned on N_T, has a binomial $(2N_T, p_t)$ distribution.

Now, to complete the calculation of the PDF of Z, we need to calculate p_{BS}. As an approximation, consider the case where there is no shadowing loss, and that a transmitter searches among the nodes that are closer to the BS and selects the one whose link loss is minimum. In this case, the probability that a node at distance r_0 from the center of the cell transmits directly to the BS in that cell, $p_{\mathrm{BS}}(r_0)$, is the same as the probability that the node cannot find a router in that region. Thus

$$\begin{aligned} p_{\mathrm{BS}}(r_0) &= \exp\left(-4\lambda_{\mathrm{Rx}} \int_0^{\frac{r_0}{2}} \sqrt{2r_0 x - x^2}\, dx\right) \\ &= \exp\left(-1.23\lambda_{\mathrm{Rx}} r_0^2\right) = \exp\left(-C\lambda_{\mathrm{Rx}} r_0^2\right) \end{aligned} \tag{31}$$

where $C = 1.23$, and p_{BS} is calculated readily by integration over r_0.

And finally, from the previous section, it still remains to calculate parameter l. To calculate l in $\Lambda_p(r_0)$, we use the fact that for a low probability of packet loss, the mean of the total intracell interference must equal the average number of generated packets per cell in each time slot. We assume that the density of packets at routers located at different distances from the center of the cell has the same distribution with different averages.

Approximating a cell by a circular region with radius r_c, the average number of packets destined to the BS which are transmitted from routers is obtained as

$$\begin{aligned} &\frac{\lambda_R}{2\lambda_{\mathrm{BS}}} \frac{\lambda_T p_t}{2\lambda_{\mathrm{BS}}} \int_0^{r_c} \exp\left(-C\lambda_{\mathrm{Rx}} r_0^2 (1+l)\right) \frac{2r_0}{r_c^2} dr_0 \\ &\propto \frac{1}{4\lambda_{\mathrm{BS}}^2} \lambda_R \lambda_T p_t \frac{1}{r_c^2 C\lambda_{\mathrm{Rx}}(1+l)} \text{ since } \exp\left(-Cr_c^2 l\lambda_{\mathrm{Rx}}\right) \\ &\propto 0. \end{aligned} \tag{32}$$

Also, the average number of packets destined to the BS which are transmitted from terminals is approximately equal to $(\lambda_T p_t/\lambda_{\mathrm{BS}})(1/C\lambda_{\mathrm{Rx}} r_c^2)$. This implies

$$\begin{aligned} \frac{\lambda_T p_t}{\lambda_{\mathrm{BS}}} &= \frac{p_t \lambda_T \lambda_R}{4\lambda_{\mathrm{BS}}^2} \frac{1}{\frac{C\lambda_R(1+l) r_c^2}{2}} + \frac{\lambda_T p_t}{\lambda_{\mathrm{BS}}} \frac{2}{C\lambda_R r_c^2} \\ \Longrightarrow l &= \frac{\frac{\lambda_R}{2} - \frac{C\lambda_R}{\pi} + 2\lambda_{\mathrm{BS}}}{\frac{C\lambda_R}{\pi} - 2\lambda_{\mathrm{BS}}} \Longrightarrow l \to_{\lambda_R \to \infty} \frac{\frac{1}{2} - \frac{C}{\pi}}{\frac{C}{\pi}} \\ &= 0.28. \end{aligned} \tag{33}$$

For the MPL case, we use an approximate model based on a combination of Poisson random variables. In this strategy, the number of terminals and the number of routers that are transmitting directly to the BSs are Poisson random variables with parameters $\lambda_t = (\lambda_T/\lambda_R/2 + \lambda_{\mathrm{BS}})$ and $\lambda_r = (\lambda_R/2/\lambda_R/2 + \lambda_{\mathrm{BS}})$, respectively. Therefore, the density of packets at a given BS, $\Lambda_p^{(\mathrm{BS})}$, can be written as

$$\Lambda_p^{(\mathrm{BS})} = \sum_{i=1}^{N} \Lambda_{p,i}^{(R)} + M \tag{34}$$

where $\Lambda_{p,i}^{(R)}$'s are the density of packets at routers which are transmitting to the BS, and N and M are Poisson random variables with parameters $\lambda_r = (\lambda_R/2/\lambda_R/2 + \lambda_{\mathrm{BS}})$ and $\lambda_t = (p_t\lambda_T/\lambda_R/2 + \lambda_{\mathrm{BS}})$, respectively. As has already been discussed, in this case, N and $\Lambda_{p,i}^{(R)}$'s are quite dependent.

To find a simple result, in the right side of (34), we assume that $\Lambda_p^{(R)}$ has a Poisson distribution with parameter $\lambda_p = (p_t\lambda_T/\lambda_{\mathrm{BS}})$, which is the average number of packets at each router. Now assuming that $\Lambda_{p,i}^{(R)}$'s, N, and M are independent random variables, the MGF of $\Lambda_p^{(\mathrm{BS})}$ can be obtained as

$$\begin{aligned} E\left(z^{\Lambda_p^{(\mathrm{BS})}}\right) &= E\left(z^{\sum_{i=1}^{N} \Lambda_{p,i}^{(R)}}\right) E(z^M) \\ &= E\left(E\left(z^{\sum_{i=1}^{N} \Lambda_{p,i}^{(R)}} | N = n\right)\right) E(z^M) \\ &= E\left(E\left(z^{\Lambda_p^{(R)}}\right)^N\right) E(z^M) \\ &= \exp\left(\lambda_r\left(E\left(z^{\Lambda_p^{(R)}}\right) - 1\right) + \lambda_t(z-1)\right) \\ &= \exp\left(\lambda_r\left(e^{\lambda_p(z-1)} - 1\right) + \lambda_t(z-1)\right). \end{aligned} \tag{35}$$

The moments of $\Lambda_p^{(\mathrm{BS})}$ can be calculated by differentiation with respect to z.

V. Analytical and Simulation Results

A. Summary of Assumptions

In this subsection, we have listed the major assumptions made for mathematical tractability. We have discussed during the course of analysis that all these assumptions are heuristically justifiable. In addition, as will be seen in the simulation results, these assumptions do not invalidate the derived closed-form formulas.

- Intercell interference due to transmissions from different nodes are considered independent.
- The density of routers is much larger than the density of BSs, so that the total received power at any BS is mainly due to packet transmissions from nodes in the same cell.
- The probability densities of intercell interference due to transmissions from terminals and routers have been approximated by normal and log-normal densities, respectively.
- The density of packets at routers located at different distances from the center of the cell has the same distribution with different averages. This average is assumed to be a fixed value in the MPL case and an exponentially decreasing function of distance from the cell center in the MFP case.
- The number of packets transmitted on terminal-BS links and the number of packets transmitted on router-BS links are independent.
- Density of packets at each receiver is calculated for small probability of error.

B. Simulation Model

For the simulation environment, we have considered a regular mesh of 50 hexagonal cells with sides of size unity. 50 BSs are located at the center of the hexagons and the performance parameters are measured at the central cell. All the other densities in the graphs are normalized to the number of BSs. For each network topology, we run the system for $2\lambda_R/\lambda_{\rm BS}p_t$ time slots before we assume a steady state has been reached. Once in the steady state, $1/p_t$ time slots are used to measure the performance parameters at the central cell. The system is rerun for new topologies, and performance parameters are updated until for each parameter, the variation in two consecutive runs is less than 1%.

C. Results

In this section, we look at the performance measures for the different forwarding strategies and investigate the accuracy of our assumptions by comparing the analytical and simulation results. The single-hop scenario and the effect of shadowing on the accuracy of our model has been discussed in [20].

The effect of shadowing can be both constructive and destructive. On one hand, it may result in a higher transmit power level on the desired link causing higher interference for other links, while on the other hand, it may block the undesired signals at certain receivers, resulting in a better decoding performance. In other words, it may result in better spatial diversity available through multiroute.

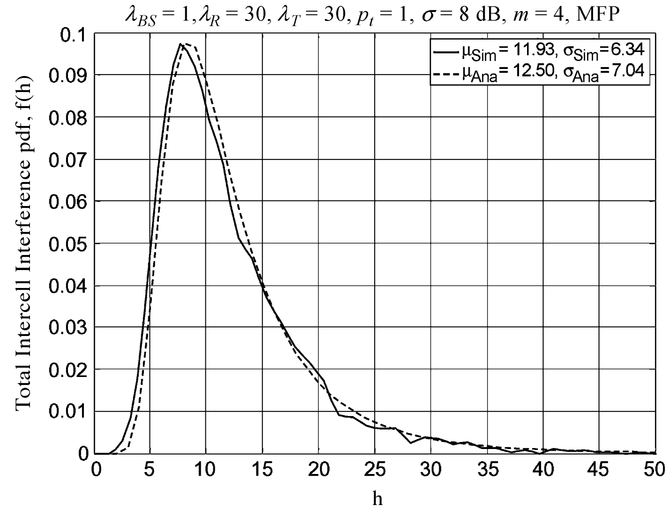

Fig. 5. (Ana)lytical and (Sim)ulation results for the pdf of the total intercell interference when routers are present.

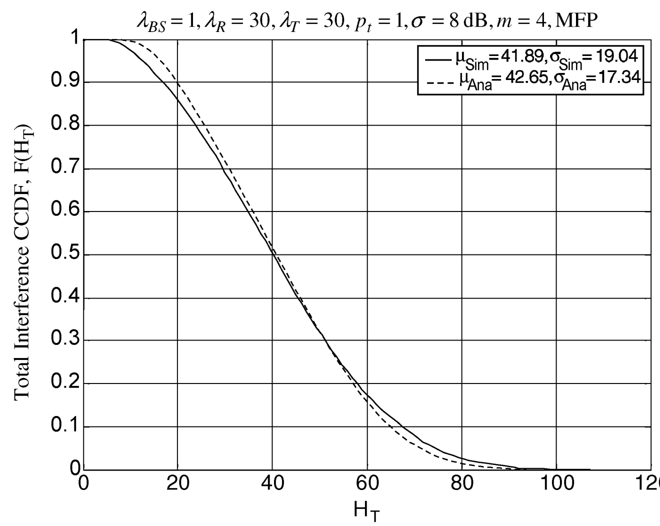

Fig. 6. (Ana)lytical and (Sim)ulation results for the CCDF of the total intercell interference when routers are present.

As we observed in the derivation of the interference pdf in the MPL case, the results were not a function of shadowing. However, note that the validity of the independence assumption is still related to the shadowing variance. For higher values of the shadowing variance, the received power levels from different nodes are less correlated, and so the variance of interference is lower. In other words, for more severe shadowing, system performance improves conditioned on perfect link-loss information being available to the transmitter. This result is also valid for the MFP case, as was seen in the derivation of the interference pdf.

A similar observation has been reported in [10] where the authors show that in a pure ad hoc network with slow fading, channel-adaptive routing techniques can benefit from the variation of fading losses over different channels to route traffic along a higher capacity path, achieving a type of spatial waterfilling with better overall throughput.

We first compare the analytical and simulation results for the pdf and complementary cumulative distribution function (CCDF) of the total intercell interference. Figs. 5 and 6 show

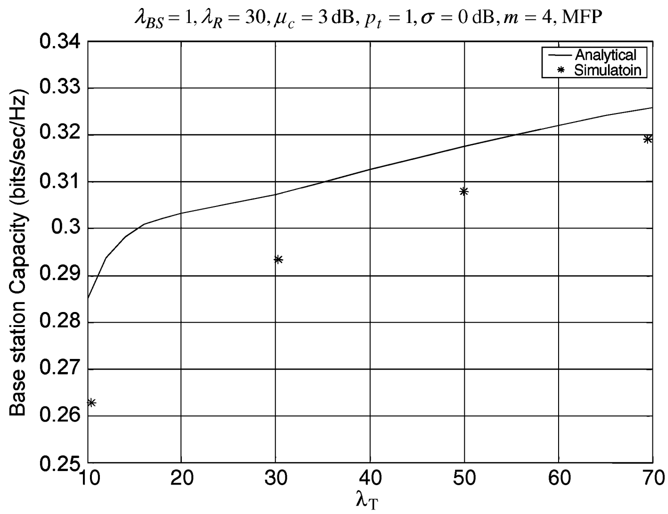

Fig. 7. Error in network capacity per cell for different densities of terminals. The results tend to become more accurate as density of terminals increases.

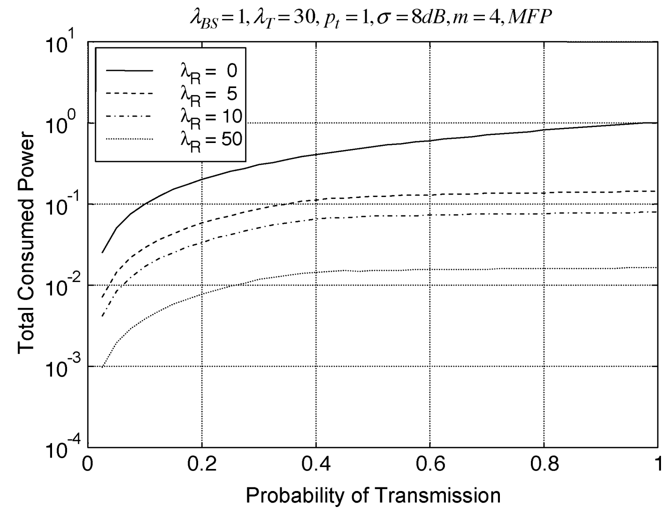

Fig. 9. Normalized total consumed power for different values of router density.

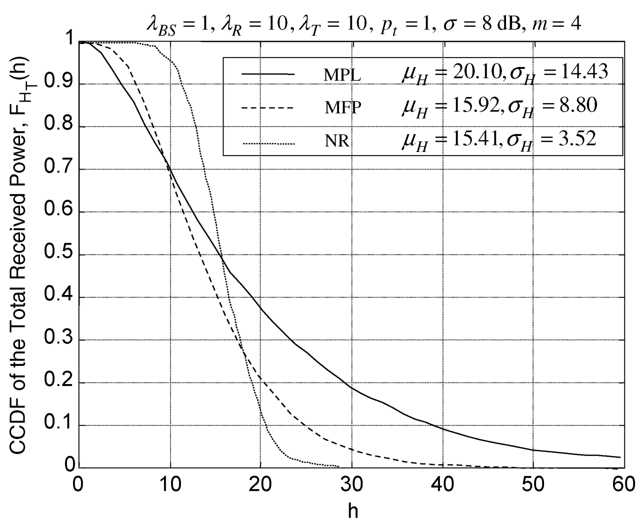

Fig. 8. Performance comparison between a conventional cellular network and a multihop cellular network. Randomly located routers to the structure increases the tail of the received power distribution.

that the error is less than 10% in the calculation of mean and standard deviation of the intercell interference.

Fig. 7 shows the error in BS throughput, γ, as defined in (11) for different terminal densities. For each value of λ_T, we obtain the required processing gain L to keep the outage probability, as defined in (13), equal to 0.01. It is seen that for higher densities of terminals, results are more accurate. For low densities, the network is underloaded and has lower throughput.

We compare in Fig. 8 the simulation results for the CCDF of the total interference at a BS for the three cases of MPL, MFP, and NR (no routers). As seen in the figure, the average interference for the two cases of MFP and NR are almost equal; however, due to randomness in locations of routers and also variance in the number of packets which routers relay, MFP has much higher interference variance. In other words, the throughput of the cellular network without using routers is better than in the other two cases. One way of comparing the throughput for these three cases is to calculate the required processing gain L for a given outage probability. To achieve a fixed outage probability of 0.01 for these three cases, when μ_c is equal to 3 dB, processing gain L in MFP and MPL must be 27% and 120% higher than the NR case, resulting in the same percentage reduction in network throughput.

In Fig. 9, we compare total system power consumption as a function of probability of transmission p_t for different values of λ_R. Case $\lambda_R = 0$ is for the structure without routers, and case $\lambda_R = 50$ is a good approximation for the structure where all terminals have the capability to retransmit the data. Power consumption for the case of all routers in comparison with the case of no routers is lower by a factor of 40. Using capacity-enhancing techniques at the physical, link, and network layers, we can benefit from the reduced transmit power due to multihop operation and further trade off power for additional capacity. For example, using higher efficient links for connections between routers and BSs will decrease the excess interference due to relaying packets, and routers act like stand-alone BSs, resulting in a capacity increase proportional to the ratio of the density of routers to the density of BSs.

VI. Conclusions

In this paper, we have analyzed and modeled the upstream achievable throughput in a multihop packet CDMA network with cellular overlay for two forwarding scenarios. The pdf of the total received power has been obtained based on the intercell and intracell interference contributions. We observe that the probability density of intercell interference due to transmissions from terminals and routers may be approximated by normal and log-normal functions, respectively. In both the MPL and MFP forwarding scenarios, assuming ideal power control, log-normal shadowing improves the system throughput and also results in lower error in analysis. In both the MPL and MFP cases, adding routers to the cellular CDMA network results in the reduction of the network throughput due to a more random structure. Although the throughput is reduced slightly, introducing routers without increasing the complexity of receivers/transmitters yields multifold savings in the system power consumption. We have illustrated that adding randomly located routers in the current cellular networks with a density of 10 routers per cell, MFP can achieve an order of magnitude in power saving at the cost of reducing the throughput only by 27%.

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Ali Nabi Zadeh (S'96–M'04) received the B.S. and M.S. degrees in electrical engineering from Sharif University of Technology, Tehran, Iran, and the University of Tehran, Tehran, Iran, in 1993 and 1999, respectively, and the Ph.D. degree in information technology from George Mason University, Fairfax, VA, in 2002.

He has worked in the communications and software engineering industry as consultant and manager for over 10 years. Currently, he is with Isocore Corporation, Reston, VA. His research interests are in performance evaluation and modeling of communication systems, Internet routing and traffic engineering, coding theory, and ad hoc networking.

Bijan Jabbari (S'80–M'81–SM'85) received the Ph.D. degree from Stanford University, Stanford, CA, in electrical engineering.

He is currently a Professor of Electrical Engineering at George Mason University, Fairfax, VA, and an affiliated faculty with ENST, Paris, France. He continues research on multiaccess communications and high-performance networking.

Dr. Jabbari is an International Division Editor for Wireless Communications of the *Journal of Communications and Networks*, an Editor for the IEEE TRANSACTIONS ON COMMUNICATIONS, and was on the Editorial Board of PROCEEDINGS OF THE IEEE. He is the past chairman of the IEEE Communications Society technical committee on Communications Switching and Routing. He was a recipient of the IEEE Millennium Medal in 2000 and the Washington, DC, Metropolitan Area Engineer of the Year Award in 2003.