# Game-Theoretic Delay-Sensitive Multirate Power Control for CDMA Wireless Networks with Variable Path Loss

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***Abstract*— A stochastic game-theoretic framework for calculating transmit power strategies for nodes in a competitive CDMA wireless network is presented. Unlike traditional game-theoretic power control, delay sensitivity is explicitly included. Stationary Nash equilibria that are a function of congestion and that allow for multirate transmission are determined. In both static path loss and dynamic shadowing path loss environments, the stochastic game equilibria calculated compare very favorably to traditional single-agent and often approach the performance of centralized optimization.**

## I. Introduction

Power control for CDMA wireless systems is an imperative, not only to mitigate the near-far problem, but also to improve the capacity of systems with scarce bandwidth. Recently, noncooperative game theory has emerged as the appropriate framework within which to analyze resource allocation problems in competitive wireless networks [1]–[4]. Such networks are presumed to consist of nodes which cannot form binding contracts of behavior and, as such, can be considered selfish utility maximizers. Research following this approach has most often dealt with the static allocation of transmit powers in networks without regard to buffer level variation in time (congestion) or to evolving channel conditions. (A notable exception is [5].) The present effort addresses both of these shortcomings and offers novel solutions.

This work applies *stochastic* game theory, which models the dynamism of the cellular uplink power-control problem by assigning a vector of buffer states to the nodes and captures the statistics of packet arrivals from higher layers and packet departures via successful transmissions, neither of which is deterministic. By adapting a recently-devised solution technique of Herings and Peeters [6], consistent stationary Nash equilibria of the delay-sensitive power control problem which map congestion level to a transmit power level are determined. Such equilibria maximize the expected utility (now and discounted in future slots) at each node. This approach represents a game-theoretic refinement of other congestion-sensitive wireless resource allocation efforts such as [7], [8].

Furthermore, evolving channel conditions can be accommodated within the preceding framework heuristically. Specifically, approximate Nash equilibria can be determined for varying channel shadowing conditions by generating and switching between multiple Nash equilibria transmit power profiles according to the observed path loss at each node. By applying this solution, path loss-adaptive game-theoretic power control can be achieved, and performance gains can be realized, without having to incorporate explicit channel transition dynamics.

The remainder of this paper proceeds as follows. Section II presents the system model, and Section III details the stochastic game formulation, the equilibrium concept, and the method of solution. Section IV presents and discusses simulation results which compare the performance of the current scheme to traditional optimization methods. Section V concludes and offers some suggestions for future work.

## II. System Model

We consider the uplink of a traditional, $n$-node CDMA wireless network. Time is divided into equal-length slots of duration $T_s$, and nodes are assumed to be in adequate synchronization. Each node contains a buffer of size $B$ bits, and in each slot, information arrives for transmission in packets of length $b$ bits according to a stationary probability mass function $p_A(\cdot)$ with positive support $1, 2, \ldots, a_{max}$.

As stationary power control strategies are sought, the transmit power of node $i$ is assumed to be a function, not of time, but exclusively of the “state” of the system (to be defined precisely in Section III). The feasible set of transmit power levels $\{P_1, \ldots, P_m\} \triangleq \mathcal{P}$, however, is independent of state and is assumed identical for all nodes.

We assume the ability via variable spreading gain to transmit more than one packet per slot, with $K$ total bit rates possible. Let $\mathcal{R} = \{R, 2R, \ldots KR\}$ be the set of available bit rates, where $R$ is the lowest possible rate. If we set $T_s = b/R$, $k$ packets can be transmitted per slot if rate $R_k = kR$ is chosen.

Given the $n$-element vector of selected powers $\mathbf{p} = (p_1, \ldots, p_n)$ and the bit rate $r_i$ and uplink path loss $\ell_i$ for node $i$, the signal-to-interference-plus-noise ratio per bit

(SINR) for node $i$ is given by:

$$\gamma_i(\mathbf{p}, r_i) = \frac{W_s}{r_i} \cdot \frac{p_i/\ell_i}{\sum_{j=1,j\neq i}^{n} p_j/\ell_j + N_r}, \tag{1}$$

where $W_s$ is the system bandwidth, and $N_r$ is the common receiver noise power.

In addition, given a probability of bit error $e_b(\gamma_i(\mathbf{p}, r_i))$, the probability that all of the transmitted bits in a packet are received correctly is assumed to be:

$$P_c(\mathbf{p}, r_i) = (1 - e_b(\gamma_i(\mathbf{p}, r_i)))^b . \tag{2}$$

We assume that transmitted packets are received independently of each other, and so the number of correctly received packets per slot is a binomial random variable.

In addition, each node is assumed to have a target packet error rate $P_e$, but due to the nature of the solution method, there is no guarantee that $P_e$ is feasible for any node. As will be formulated below, the multirate operation is assumed to be *implicit:* given $\mathbf{p}$, each node determines the bit rate that yields the highest expected throughput, with the constraint that $P_e$ is satisfied. If no bit rate satisfies $P_e$, then the lowest bit rate $R$ is used.

Path losses and other system parameters not explicitly singled out are assumed to be common knowledge across the nodes. Notably, although it could pose practical problems, it is assumed that buffer levels of the terminals are common knowledge.

## III. STOCHASTIC GAME FORMULATION AND EQUILIBRIUM EXISTENCE

### A. Static Path Loss Assumption

*1) Game Formulation:* A finite, discounted stochastic game $\Gamma$—with present simplifications—is given by the following six-tuple [6]:

$$\Gamma = \left\langle N, \Omega, \mathcal{P}, \{u_i\}_{i\in N}, \pi, \delta \right\rangle . \tag{3}$$

1) $N = \{1, \ldots, n\}$ is the set of mutually interfering nodes.
2) $\Omega = \{\omega_1, \ldots, \omega_z\}$ is the state space for the system, here the set of possible buffer levels (in units of $b$-bit packets) across the $n$ transmitting nodes. We assume that $\Omega$ is "vectorized" into substates, i.e., there exists a mapping $\psi : \Omega \to \Omega'$, where $\Omega'$ is a set of $n$-tuples with cardinality $\mathrm{card}(\Omega)$. Specifically, we assume that $\Omega' = \times_{i=1}^{n} W$, where $W = \{1, 2, \ldots, z^{1/n} = B/b\}$ is the (identical) substate for each node. $W$ is thus the set of possible buffer levels for each node.[1] A typical member of $\Omega'$ could thus be written as $(\omega'_1, \omega'_2, \ldots, \omega'_n)$, where $\omega'_i$ is the substate for node $i$. In the simplified case of $n = 2$, $\psi(\omega)$ is given by the mapping:

$$\omega'_1 = 1 + \lfloor(\omega - 1)/\sqrt{z}\rfloor \tag{4a}$$
$$\omega'_2 = 1 + (\omega - 1) \bmod \sqrt{z} \tag{4b}$$

for any $\omega \in \Omega$. Thus for $z = 9$, $\psi(7) = (3, 1)$. An analogous mapping for $n > 2$ is easily derived.
3) $\mathcal{P}$ is the aforementioned static feasible set of transmit powers for all nodes and states.
4) $u_i : H \to \mathbb{R}$ is the (deterministic) per-slot payoff function of node $i \in N$, where $H \triangleq \{(\omega, \mathbf{p}) \,|\, \omega \in \Omega, \mathbf{p} \in \mathbf{P}\}$ and $\mathbf{P} \triangleq \times_{i=1}^{n} \mathcal{P}$ is the joint action space of the $n$ nodes. Many utility functions are conceivable, but we prefer to employ a utility function that captures the appropriate incentives while being simple and intuitive. Our starting point is the utility employed in [2], [9] which seeks to express the utility in units of expected number of bits transmitted per Joule of expended transmit power. Ignoring any framing bits, this utility is expressed in the present notation as follows:

$$u_i(\mathbf{p}) = \frac{k_{opt} R P_c(\mathbf{p}, R_{k_{opt}})}{p_i} \; \frac{\text{bits}}{\text{Joule}}, \tag{5}$$

where $P_c(\cdot)$ is given in (2).[2] In (5), $k_{opt}$ is the optimal mode, given by:

$$k_{opt} = \begin{cases} \underset{k\in\mathcal{K}_f}{\arg\max}\{k P_c(\mathbf{p}, R_k) : P_c(\mathbf{p}, R_k) \geq 1 - P_e\}, \\ \qquad\qquad \text{if } P_c(\mathbf{p}, R) \geq 1 - P_e; \\ 1, \qquad\quad \text{otherwise,} \end{cases} \tag{6}$$

where $\mathcal{K}_f = \{1, \ldots, \min\{\omega'_i, K\}\}$.

Now, in order to make this utility state-dependent and thereby provide an incentive for each node to empty its buffer, we modify (5) so that the utility is the expected *fraction of the buffer emptied* per Joule of expended transmit power. We are thus led to the following utility function:

$$\begin{aligned} u_i(\omega, \mathbf{p}) &= u_i(\omega'_i, \mathbf{p}) \\ &= \frac{k_{opt} R P_c(\mathbf{p}, R_{k_{opt}})}{b\,\omega'_i p_i} \; \frac{\%\text{ of buffer}}{\text{Joule}}. \end{aligned} \tag{7}$$

5) $\pi : H \to \Delta(\Omega)$ are the state transition probabilities, where $\Delta(\Omega)$ is the set of all probability distributions on $\Omega$. A key driving principle in this section is that, since the utility function must be deterministic, the transition function must account for *both* probabilistic packets arrivals from higher layers and departures due to successful transmissions. Let $\omega^*$ be the potential next state. Note that the transition probabilities, conditioned on the current state $\omega$ and power vector $\mathbf{p}$, are independent from node to node, and thus, since the state space is vectored, we may decompose the transition probabilities

[1] It may be noticed that in the present formulation the buffer is assumed to never empty. This is done for convenience, but it is not essential to this method.

[2] In [2], [9], the packet success function was modified slightly to prevent the zero transmit power case from yielding an infinitely high utility. We may ignore this as $\mathcal{P}$ is sufficiently bounded. Also of interest is that the authors in [2] assert that that same factor amounted to an implicit delay constraint. In what follows, an *explicit* delay constraint is added.

$\pi(\omega^* \,|\, \omega, \mathbf{p})$ in the natural way:

$$\pi(\omega^* \,|\, \omega, \mathbf{p}) = \prod_{i=1}^{n} \pi_i(\omega_i^{*\prime} \,|\, \omega, \mathbf{p}), \qquad (8)$$

where $\omega_i^{*\prime}$ is the $i$th component of the vectored state $\omega^{*\prime}$. The transition probability for node $i$ may be in turn decomposed as follows:

$$\begin{aligned}
&\pi_i(\omega_i^{*\prime} \,|\, \omega, \mathbf{p}) \\
&= \pi_i(\omega_i^{*\prime} \,|\, \omega_i', \mathbf{p}) \\
&= \sum_{a=1}^{a_{max}} \sum_{\tau=1}^{\omega_i'} \mathbb{P}\{\omega_i^{*\prime} \,|\, \omega_i', \mathbf{p}, T=\tau, A=a\} \\
&\qquad \cdot \mathbb{P}\{T=\tau \,|\, \omega_i', \mathbf{p}, A=a\} \cdot \mathbb{P}\{A=a\} \\
&= \sum_{a=1}^{a_{max}} \mathbb{P}\{A=a\} \sum_{\tau=1}^{\omega_i'} \mathbb{P}\{T=\tau \,|\, \omega_i', \mathbf{p}\} \\
&\qquad \cdot \mathbb{P}\{\omega_i^{*\prime} \,|\, \omega_i', \mathbf{p}, T=\tau, A=a\}. \qquad (9)
\end{aligned}$$

We consider each factor of (9) in turn.

a) $\mathbb{P}\{A=a\}$ is simply the static PMF of the packet arrival process, $p_A(\cdot)$.

b) $\mathbb{P}\{\omega_i^{*\prime} \,|\, \omega_i', \mathbf{p}, T=\tau, A=a\}$ is the probability that given the current state, power vector, and number of transmitted and arrival packets, the state $\omega_i^{*\prime}$ is attained. There are three subcases to examine here, but these are omitted for brevity.

c) $\mathbb{P}\{T=\tau \,|\, \omega_i', \mathbf{p}\}$ is the probability that $\tau$ packets are transmitted given the current state and power vector. Since the modal selection is deterministic, this term will either be one or zero. Specifically:

$$\mathbb{P}\{T=\tau \,|\, \omega_i', \mathbf{p}\} = \begin{cases} 1, & \text{if } \tau = k_{opt}; \\ 0, & \text{otherwise,} \end{cases} \qquad (10)$$

where $k_{opt}$ is determined according to the optimization in (6).

Therefore, from (8), we have the final expression for the transition probabilities:

$$\begin{aligned}
\pi(\omega^* \,|\, \omega, \mathbf{p}) = \prod_{i=1}^{n} \Bigg[ &\sum_{a=1}^{a_{max}} p_A(a) \sum_{\tau=1}^{\omega_i'} \mathbb{P}\{T=\tau \,|\, \omega_i', \mathbf{p}\} \\
&\cdot \mathbb{P}\{\omega_i^{*\prime} \,|\, \omega_i', \mathbf{p}, T=\tau, A=a\} \Bigg]. \qquad (11)
\end{aligned}$$

6) $\delta \in (0,1)$ is the discount factor for future payoffs and would normally be assumed near unity for nodes whose termination horizons are in the indeterminate future [10].

*2) Equilibrium:* The objective is to find a consistent equilibrium strategy for all nodes given the game definition in Section III-A.1. We narrow our consideration to strategies which are *stationary,* i.e., those strategies which are independent of the history of the game and of time and thus depend solely on the current state of the system. We define a stationary strategy formally as follows [6].

*Definition 1:* A *stationary strategy* $\rho_i$ for node $i$ is given by:

$$\rho_i = \left(\rho^i_{\omega_1}, \ldots, \rho^i_{\omega_z}\right) \qquad (12)$$

where $\rho^i_\omega$ is a probability measure on $\mathcal{P}$ for each $\omega \in \Omega$.

*Definition 2:* A stationary strategy $\rho_i$ is said to be a *pure* strategy if, for each state $\omega$, the optimal transmit power is deterministic. Otherwise, $\rho_i$ is said to be *mixed.*

An equilibrium strategy-tuple $\rho = (\rho_1, \ldots, \rho_n)$ is a vector of stationary strategies which possesses the Nash equilibrium property that no node has an incentive to unilaterally deviate from its strategy. Such equilibria could thus be said to exhibit competitive stability.

A classical result of game theory is that there exists at least one—possibly mixed—stationary equilibrium to every stochastic game [11], but calculating equilibrium strategies and guaranteeing that all nodes will converge to the same equilibrium are traditionally difficult problems.

In [6], Herings and Peeters present a new algorithm for calculating equilibrium strategy-tuples for discounted stochastic games. This algorithm, which employs a stochastic tracing procedure that extends seminal work of Harsanyi and Selten [12], offers the following: if a common, pure initial strategy (or *prior*) is assumed, the algorithm yields a consistent (possibly mixed) equilibrium across the nodes.[3] It is also a globally convergent algorithm, but there is no guarantee of speedy convergence.

### B. Dynamic Path Loss

In Section III-A, the system path losses are assumed to be static, but we may extend the formulation to account for a *dynamic* path loss. Most generally, path loss would be included in the state space, and the dynamics of the path loss would be accounted for in the transition probabilities. Due to inherent complexity, though, this is impractical. However, *approximate* equilibrium strategies in a slow, shadow fading environment can be obtained. (In what follows, short-term fading is assumed to be mitigated.)

Let $\{\ell_1, \ldots, \ell_q\} \triangleq \mathcal{L}$ be the set of (quantized) path loss values realizable by each node due to shadowing. Furthermore, let $\mathbf{L} = \times_{i=1}^{n} \mathcal{L}$ be the vector of possible path loss values with typical value $L \in \mathbf{L}$. Define the mapping $\phi : \mathbf{L} \to \mathbf{R}$, where $\mathbf{R}$ is the set of all equilibrium strategies, by $\phi(L) = \rho_L$, where $\rho_L$ is the equilibrium strategy-tuple obtained with static path loss vector $L$. As the observed path loss evolves, $\phi$ determines the new strategy to be used by all nodes.

We acknowledge that in this case strategies will not strictly be Nash equilibria because the dynamics of the path loss evolution are not included in the formulation. However, since future payoffs are discounted and the shadowing coherence time is expected to be long compared to the slot time, we may realistically expect that the loss in optimality will not be

[3]We note that although mixed strategies are possible in this algorithm, our experience has shown that pure strategies are nearly universally realized.

very great. An additional advantage of this approach is that the strategies may be calculated *a priori* and switched among during network operation.

## IV. Simulation and Discussion

In order to evaluate the efficacy of the current approach, the game formulations of Section III were simulated in MATLAB, and we examine the simple case of $n = 2$. The stationary equilibrium strategies themselves, given the common prior strategy and the utilities and transition probabilities according to (7) and (11), respectively, were generated using the StochasticGameSolver Fortran code [13]. For the common prior, a reasonable assumption is for each node to transmit at the highest possible power $P_m$ at the onset. However, we also made use of additional priors: all nodes transmitting at (or approximately at) the lowest power $P_1$, nodes adopting the single-agent strategy (described below), or nodes using a random—but consistent—strategy. Depending on the system parameters, some priors we seen to realize speedier convergence to equilibrium than others, and such speedier priors were adopted on a case-by-case basis. The modulation is assumed to be BPSK, and therefore $e_b(\gamma_i) = Q\left(\sqrt{2\gamma_i}\right)$ [14].

For comparison, stationary single-agent and centralized optimization strategies were also calculated. Traditional single-agent optimization involves evaluating an infinite-horizon, discounted dynamic programming problem by solving the Bellman equation [15]:

$$V_i(\omega_i') = \max_{p_i \in \mathcal{P}} \left\{ u_i(\omega, \mathbf{p}) + \delta \sum_{\omega' \in W} \pi_i\left(\omega_i^{*\prime} \,\middle|\, \omega_i', \mathbf{p}\right) V_i(\omega_i') \right\}, \tag{13}$$

for all $\omega_i' \in W$ and for all $i \in N$. Since there is no strategizing in this case, the transmit powers of the neighboring nodes must be assumed. A reasonable, yet pessimistic, assumption is to assume that neighboring nodes transmit at the highest available power.

The centralized optimization of $n$ nodes, which is expected to yield the utility-maximizing allocation of transmit powers, is given in like manner to (13), albeit in $n$ dimensions. For both the single-agent and centralized $n$-node optimization, code from the CompEcon Toolbox [16] was employed for convenience.

### A. Static Path Loss

For the static path loss case, an example of the total discounted utilities realized by the nodes over 100 time slots using the game-theoretic, centralized, and single-agent strategies (and combinations thereof) is shown in Fig. 1. The utilities are presented normalized to the centralized solution, and we observe a steady improvement with the employment of the game-theoretic over the single-agent approach. In this particular case, the performance of the game-theoretic strategy matches identically that of the centralized allocation for each trial.

The average total discounted utility over 100 slots for the two nodes is shown in Table I as a function of the nodes' path loss. We see that in nearly every case, the game-theoretic equilibrium achieves the centralized utility, an atypically efficient outcome [17]. The game-theoretic solution also exhibits a 22% to 98% improvement in total utility over the single-agent case. In addition, as expected, we observe a general improvement in utilities with lower path losses; nodes find it comparatively easier to empty the buffer under more favorable channel conditions.

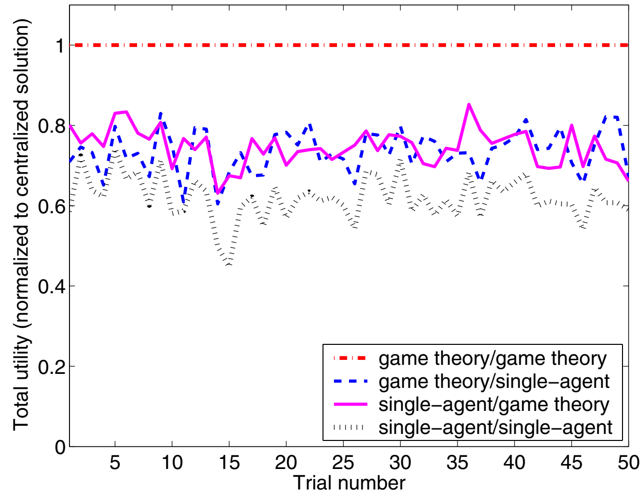


Fig. 1. An example of the total utility realized over 100 slots (normalized to the centralized solution) for 50 trials with $(\ell_1, \ell_2) = (110\text{ dB}, 110\text{ dB})$. The legend indicates the respective approaches used by the two nodes. Parameters used in this and other simulations: $m = 4$, $P_1 = -35$ dBW, $P_m = -25$ dBW, $\mathcal{P}$ is uniformly distributed in dB, $P_e = 10^{-3}$, $W_s = 1.25$ MHz, $R = 20$ kbps, $N_r = -143$ dBW, $a_{max} = 5$, $p_A(1) = 0.30$, $p_A(2) = 0.25$, $p_A(3) = 0.20$, $p_A(4) = 0.15$, $p_A(5) = 0.10$, $\delta = 0.95$, $K = 4$, $b = 120$ bits, $z = 9$, $q = 3$, and $\mathcal{L} = \{100\text{ dB}, 105\text{ dB}, 110\text{ dB}\}$ with 115 dB used in some tests.

TABLE I
Average total discounted utility over 100 slots realized via the various power control strategies (static path loss). Data were averaged over 50 trials with randomized initial state.

| | Average total utility (% of buffer per $\mu$J) | | |
|---|---|---|---|
| $(\ell_1, \ell_2)$ (dB) | Single-agent | Game theory | Centralized |
| $(100, 100)$ | 11.93 | 20.96 | 20.96 |
| $(100, 105)$ | 14.01 | 17.08 | 17.08 |
| $(100, 110)$ | 11.53 | 14.54 | 14.54 |
| $(105, 105)$ | 10.59 | 20.96 | 20.96 |
| $(105, 110)$ | 9.34 | 14.19 | 17.07 |
| $(105, 115)$ | 11.53 | 14.07 | 14.07 |
| $(110, 110)$ | 10.49 | 16.91 | 16.91 |

In addition to improving the total utility—as designed—we observe that the game-theoretic equilibria obtained generally require much less total transmit energy over single-agent optimization. Fig. 2 shows the total transmit energy expended over 100 slots relative to that expended by the centralized solution, and Table II presents the average total transmit energy over 100 slots. As with the utility, the game-theoretic equilibria

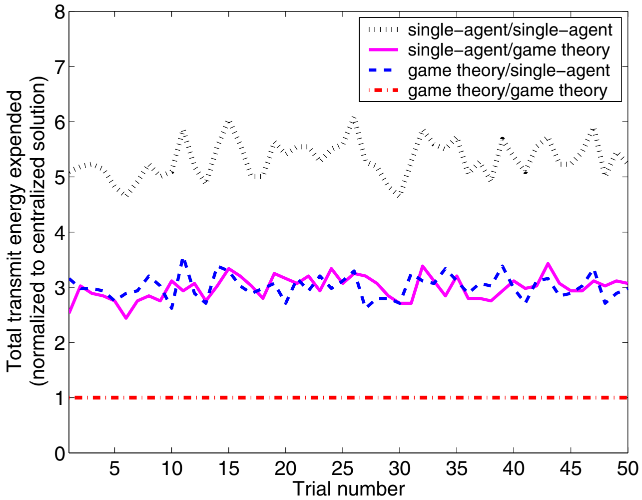


Fig. 2. An example of the total transmit energy expended over 100 slots (normalized to the centralized solution) for 50 trials with $(\ell_1, \ell_2) = (110 \text{ dB}, 110 \text{ dB})$.

TABLE II

AVERAGE TOTAL TRANSMIT ENERGY EXPENDED OVER 100 SLOTS VIA THE VARIOUS POWER CONTROL STRATEGIES (STATIC PATH LOSS). DATA WERE AVERAGED OVER 50 TRIALS WITH RANDOMIZED INITIAL STATE.

| | Average total transmit energy (mJ) | | |
|---|---|---|---|
| $(\ell_1, \ell_2)$ (dB) | Single-agent | Game theory | Centralized |
| $(100, 100)$ | 1.04 | 0.38 | 0.38 |
| $(100, 105)$ | 0.97 | 0.53 | 0.53 |
| $(100, 110)$ | 2.09 | 1.26 | 1.26 |
| $(105, 105)$ | 2.01 | 0.38 | 0.38 |
| $(105, 110)$ | 0.93 | 0.38 | 0.53 |
| $(105, 115)$ | 2.09 | 1.34 | 1.34 |
| $(110, 110)$ | 2.02 | 0.38 | 0.38 |

achieve much improved energy expenditures over the single-agent case and yield equal (and in one case slightly less) total transmit energy than the centralized optimization. Energy savings is thus successfully realized.

Although yielding satisfactory results, the algorithm used to generate the Nash equilibrium strategies above is not without its drawbacks. It was typically quite time-consuming to generate the equilibria, and convergence times were unpredictable. Most restrictive, however, was the "curse of dimensionality": we were limited to small state and action spaces as large spaces were prohibitively complex.

### B. Dynamic Path Loss

For the dynamic path loss case, a shadowing map was generated using the correlated shadowing model of [18] with a smooth interpolation between shadowing values. Nodes were able to switch strategies after each time slot based on new path loss measurements. The mapping $\phi$ from Section III-B is defined in the natural way: the combined strategy corresponding to the path loss pair closest (in an absolute-value sense) to the most recently observed path loss values is chosen.

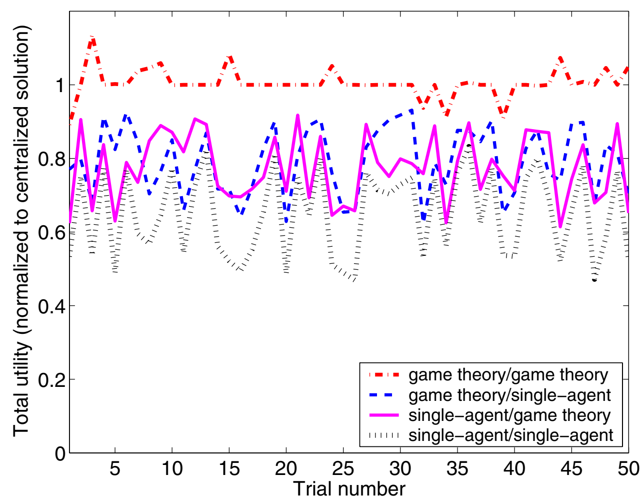


Fig. 3. Total utility realized per 100 slots (normalized to the centralized solution) for 50 trials with initial path loss of 105 dB for each node. Parameters used in the shadowing map include a cell radius of 600 m, a shadowing correlation distance of 100 m, a shadowing standard deviation of 6 dB, and node speed of 4.17 m/s. A constant starting point for each node was assigned, and the direction of travel for each node was chosen randomly from the four cardinal directions at the onset and held constant. The discounting of future payoffs was "reset" after 100 time slots.

TABLE III

AVERAGE TOTAL DISCOUNTED UTILITY PER 100 SLOTS REALIZED VIA THE VARIOUS POWER CONTROL STRATEGIES (DYNAMIC PATH LOSS). DATA WERE AVERAGED OVER 50 TRIALS WITH RANDOMIZED INITIAL STATE.

| Average total utility (% of buffer per $\mu$J) | | |
|---|---|---|
| Single-agent | Game theory | Centralized |
| 10.87 | 17.17 | 17.09 |

The initial path loss for each node was 105 dB and the system was evolved for 400 time slots. Fig. 3 demonstrates the total discounted utility relative to the centralized solution for the various strategies, and Table III shows the average total discounted utility per 100 slots. In this case, we observe that although there is an expected reduction in utility for the game-theoretic and centralized solutions (due to path loss tracking error), we see that the game-theoretic approach is generally as effective as (or even at times slightly more effective than) the centralized strategy. The game-theoretic solution also exhibits a 58% improvement in total utility over the single-agent case.

## V. CONCLUSIONS AND FUTURE WORK

A stochastic game-theoretic approach to calculating transmit power strategies for nodes in a delay-sensitive CDMA wireless network was presented. Simulation has shown that in both static path loss and dynamic shadowing path loss cases, the game-theoretic equilibria determined compare favorably to traditional single-agent and centralized optimization techniques in terms of realized utility and transmit energy expended, albeit at the cost of increased system and computational complexity.

Future work could entail refinements to the utility function, perhaps by including pricing terms for transmit power [2]. More importantly, attempts at reducing the complexity of this general framework would be effort well spent. By simplifying or coarsening the state space, or by investigating alternative and approximate equilibrium concepts, this approach may be profitably applied to richer networks. It is by employing such approximations that game-theoretic power control solutions may have their greatest practical impact.

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