Link Performance Bounds in Homogeneous Optically Switched Ring Networks

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***Abstract*— We consider a ring topology with limited or full switching capability as deployed in high bandwidth metro optical networks and develop a model to estimate the probability of blocking for interconnecting links. The model is based on the lower and upper bounds of link blocking probability. We analyze the performance for a homogeneous traffic case and present simulation results for representative ring networks. We demonstrate that the bounds are very tight with an error of less than 2% when the traffic load is modest. The approach is based on partitioning the state space into subspaces and weighting the upper bound of blocking probability in each subspace with the occurrence of the states. The computational complexity of the approach is comparable to solving a degree of N polynomial equation.**

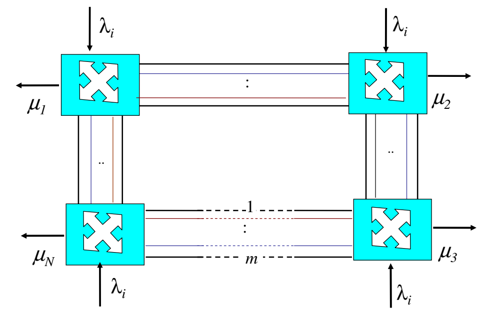

Fig. 1. Optically Switched Ring Networks

I. Introduction

Optical network elements such as Reconfigurable Optical Add Drop Multiplexers (ROADMs) and Optical Cross-Connects (OXC) are becoming integral parts of wavelength division multiplexing (WDM) networks [1]. Estimating the blocking probability of interconnecting wavelength links in a WDM network is important in assessing the Quality of Service (QoS) from the carriers' perspective or as perceived by the end users. This, in particular, is the case when the traffic intensity is so high that due to call blocking, the discrepancy between the entire offered load and the carried load is not negligible. While the exact computation of blocking probability may be unavailable, bounds may serve as a useful measure for design and provisioning purposes, especially if the upper and lower bounds are found to be very tight.

Variants of ring architecture are widely deployed by service providers. The mesh architecture may be developed by extending the existing ring topology to further simplify the interconnection of these networks at the core. In an optical network, whether the connection is part of a Traffic Engineered path or a deterministic path, each request of lightpath setup will take one wavelength. This is similar to circuit setup in a public-switched network except that the former is subject to wavelength continuity and, more generally, optical impairment constraints.

With development of a control plane or sophisticated management and provisioning systems, a dynamic set of lightpaths becomes a reality. For example, the Generalized Multi-Protocol Label Switching (GMPLS) [2], provides a unified end-to-end control plane for provisioning, protection, and restoration of heterogeneous data networks, and enables each node to do traffic engineering based on the advertised resource availability information.

In this paper, we consider a ring topology with dynamic optical links interconnecting the network nodes and develop upper and lower bounds to estimate the probability of link blocking. Next section describes the system model under consideration. Section III presents our model, followed by the derivation of the bounds in section IV and V. Section VI presents the simulation results and finally concluding remarks are provided in Section VII.

II. System Model

As shown in Fig.1, N access nodes are connected to a unidirectional optically switched ring. A fiber on the ring has m wavelengths, and each node j has the same incoming traffic λ_i. We assume that wavelength conversion is unavailable, the traffic demand matrix is homogeneous, and traffic only flows clockwise. Meanwhile, incoming traffic at each access node is a Poisson process, and the traffic is distributed among all the wavelengths randomly with equal probability. If a request cannot be satisfied on the selected wavelength plane, it is rejected. The service time of each request is assumed to follow an exponential distribution.

From the above assumptions, we know that the arrival to each wavelength plane is also a Poisson process. The problem is simplified to determining a blocking probability on each wavelength plane given the Poisson arrival rate of incoming traffic of λ_i/m at each node.

Blocking probability in various network scenarios have been addressed in [3]–[6]. The blocking probability in all-optical networks with and without wavelength changers has been considered in [3] and has been modeled with the assumption that the traffic load is light, an initial estimate of link blocking probability is known, and usage of a wavelength on a hop is statistically independent of other hops as well as other

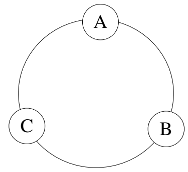

Fig. 2. A ring with three nodes

wavelengths. The estimation of fiber utilization ratio in [3] neglects the impact on call arrival rate caused by the blocking. This is reasonable when the link blocking probability is low. However, with high link blocking probability, the assumption will result in over-estimating the path blocking probability. Moreover, by assuming that wavelength seizure and release are independent of each other, the dynamic nature of the traffic is hidden.

Reference [4] focuses on the optical network with wavelength converters. The traffic is assumed to be bounded by the number of ports in a node, which hides the dynamic nature of the traffic. Reference [5] provides both analytical model and simulation results on call blocking probability in a ring network for very light traffic. This model assumes a homogeneous traffic matrix, Markovian correlation of blocking at adjacent links, and certain regular topologies. The computational complexity is modest.

More recently, a computational model for estimating blocking probability in a multi-fiber WDM optical network has been presented in [6] where the entire wavelength channel is dedicated to a single connection. Our work deals with dynamically provisioned networks, where even if a wavelength is occupied on some segments of a network, it can still be reused wherever possible, hence considerably reducing the blocking probability.

With Fiber to the Home (FTTH), the arrival rate at an edge node will become high and blocking probability may be much greater than that in the core network. Estimation of blocking probability in a network with arbitrary traffic intensity is a hard problem in that the correlation of blocking probability on different links makes precise computation of carried load impossible.

We note that link blocking probability is usually the basis to compute call blocking. This will be the focus of our paper and is equally applicable to [3] and [5]. It can further be extended to model waveband switching or fiber switching.

III. State Transition Diagram of a Unidirectional Ring Topology

A. A Simple Example

Suppose we have 3 nodes in a unidirectional ring. Each link has only one wavelength. Fig. 2 shows that both link AB and BC being busy can be caused by two scenarios, i.e., AB is occupied by a lightpath from A to B and BC is occupied by a lightpath from B to C or AB and BC is occupied by a lightpath from A to C, which means that we should not only note whether a link is busy, but also note the source-destination pair which occupies the link.

TABLE I

List of Link States

Link		Occupied by path	Occupied by path	Occupied by path
AB	1. Idle	2. From A to B	3. From A to C	4. From C to B
BC	5. Idle	6. From B to C	3. From A to C	7. From B to A
CA	8. Idle	9. From C to A	4. From C to B	7. From B to A

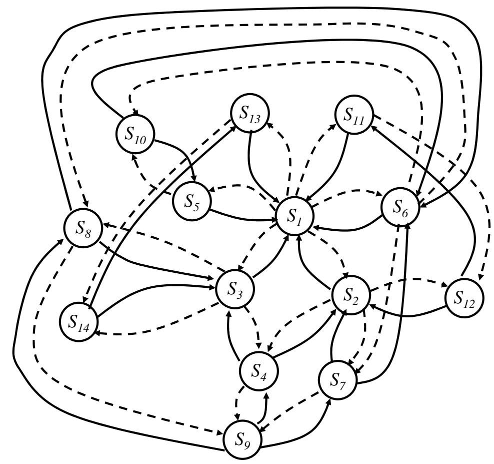

Fig. 3. State Transition Diagram of a Three-node Ring. Each arrow represents transition rate of $\lambda/2$, and each arrow with dashed line represents transition rate of μ.

We can list the states as shown in Table I. Therefore, we can find 14 different states for a three-node ring. We define S_1 as the scenario that all links are idle. S_1 can be described as $S_1 = (1, 5, 8)$, where the three indices in the parentheses are defined in Table I. Similarly, we define $S_2 = (1, 5, 9)$, $S_3 = (1, 6, 8)$, $S_4 = (1, 6, 9)$, $S_5 = (1, 7, 7)$, $S_6 = (2, 5, 8)$, $S_7 = (2, 5, 9)$, $S_8 = (2, 6, 8)$, $S_9 = (2, 6, 9)$, $S_{10} = (2, 7, 7)$, $S_{11} = (3, 3, 8)$, $S_{12} = (3, 3, 9)$, $S_{13} = (4, 5, 4)$ and $S_{14} = (4, 6, 4)$.

We assume that the traffic demand matrix is homogeneous, the incoming traffic at each node is λ, the departure rate of each lightpath request is μ. The state transition diagram of Fig. 2 can be depicted as in Fig. 3.

Based on Fig. 3, we can obtain the state transition matrix as shown in Fig. 4. We can obtain the probability of each state in the following vector:

$$S' = [\frac{8}{D}\ \frac{4r}{D}\ \frac{4r}{D}\ \frac{2r^2}{D}\ \frac{4r}{D}\ \frac{4r}{D}\ \frac{2r^2}{D}\ \frac{2r^2}{D}\ \frac{r^3}{D}\ \frac{2r^2}{D}\ \frac{4r}{D}\ \frac{2r^2}{D}\ \frac{4r}{D}\ \frac{2r^2}{D}], \tag{1}$$

where $D = r^3 + 12r^2 + 24r + 8$, and $r = \lambda/\mu$.

According to (1), we can easily calculate the probability that link AB is busy as:

$$P_b = \sum_{k=6}^{14} S_k. \tag{2}$$

B. Partitioning the state space

A three-node unidirectional ring with homogeneous traffic demand is the simplest scenario. However, calculating the

	S_1	S_2	S_3	S_4	S_5	S_6	S_7	S_8	S_9	S_{10}	S_{11}	S_{12}	S_{13}	S_{14}
S_1	-3λ	μ	μ	0	μ	μ	0	0	0	0	μ	0	μ	0
S_2	$\lambda/2$	$-\mu-1.5\lambda$	0	μ	0	0	μ	0	0	0	0	μ	0	0
S_3	$\lambda/2$	0	$-\mu-1.5\lambda$	μ	0	0	0	μ	0	0	0	0	0	μ
S_4	0	$\lambda/2$	$\lambda/2$	$-2\mu-0.5\lambda$	0	0	0	0	μ	0	0	0	0	0
S_5	$\lambda/2$	0	0	0	$-\mu-0.5\lambda$	0	0	0	0	μ	0	0	0	0
S_6	$\lambda/2$	0	0	0	0	$-\mu-1.5\lambda$	μ	μ	0	μ	0	0	0	0
S_7	0	$\lambda/2$	0	0	0	$\lambda/2$	$-2\mu-0.5\lambda$	0	μ	0	0	0	0	0
S_8	0	0	$\lambda/2$	0	0	$\lambda/2$	0	$-2\mu-0.5\lambda$	μ	0	0	0	0	0
S_9	0	0	0	$\lambda/2$	0	0	$\lambda/2$	$\lambda/2$	-3μ	0	0	0	0	0
S_{10}	0	0	0	0	$\lambda/2$	$\lambda/2$	0	0	0	-2μ	0	0	0	0
S_{11}	$\lambda/2$	0	0	0	0	0	0	0	0	0	$-\mu-0.5\lambda$	μ	0	0
S_{12}	0	$\lambda/2$	0	0	0	0	0	0	0	0	$\lambda/2$	-2μ	0	0
S_{13}	$\lambda/2$	0	0	0	0	0	0	0	0	0	0	0	$-\mu-0.5\lambda$	μ
S_{14}	0	0	$\lambda/2$	0	0	0	0	0	0	0	0	0	$\lambda/2$	-2μ

Fig. 4. State Transition Matrix in a Three-node Ring

precise link blocking probability is complicated. Generally, for a ring with N nodes, we need $O(N^N)$ network states, and since we need to compute the transition rate for any state to any other state, an $O(N^N \times N^N)$ matrix results. Even if N is small, the matrix becomes intractable due to its non-polynomial complexity. A precise state transition diagram to obtain the blocking probability is not possible when N is large.

When the traffic demand matrix is homogeneous, the blocking probability on each link is also the same. For a ring with N links denoted as $(l_1, l_2, \cdots, l_N)$, we focus on the blocking probability on l_1.

The basic idea of the proposed algorithm is straightforward. The arrival rate on l_1 is dependent on the current network states. We assume that there are K network states, and the flow rate on l_1 of state k is I_k. Without loss of generality, we assume the departure rate of all the states is one per time unit. We denote p_k as the probability that the network is in state k. The blocking probability of link l_1 is:

$$P_b = \sum_{k=1}^{K} \frac{I_k}{1 + I_k} p_k. \tag{3}$$

When all links are idle, the flow rate on l_1 is the maximum. When link l_N is busy, the flow rate on l_1 is reduced. If both l_N and l_2 are busy, the flow rate on l_1 is further reduced. We denote the network state space as Ω.

The proposed algorithm in this paper simplifies the state transition diagram by dividing Ω into 3 subspaces as follows:

$$S_0 = \Omega - S_1 - S_2. \tag{4}$$

$$S_1 = \{l_N \text{ busy and } l_2 \text{ idle}\}. \tag{5}$$

$$S_2 = \{l_N \text{ busy and } l_2 \text{ busy}\}. \tag{6}$$

We use the maximum flow rate I_{sk} among all the states $s_{km} \in S_k$ as the representative flow rate in S_k, and denote p_{sk} as the probability that the network is in state S_k, and define $P_{bu}^{Sk} = I_{sk}/(1 + I_{sk})$. Therefore, we have

$$\sum_{k=0}^{2} (P_{bu}^{Sk} p_{sk}) > P_b. \tag{7}$$

Equation (7) cannot be used directly because it is of the same complexity to calculate the precise p_{sk}. Further approximation is required.

IV. Lower Bound of Blocking Probability

Suppose the blocking probability on each link is P_b. We offer the following approach to compute the lower bound of blocking probability on a link. In all subsequent analysis, we assume the traffic matrix is homogeneous with newly generated traffic intensity of λ at each node. The ring is unidirectional and traffic flows clockwise.

Theorem 1: In a ring with N nodes, if node A is k hops away from the head node of link δ, the traffic flow $I(k)$ from A on the link δ satisfies

$$I(k) \geq \frac{\lambda((1 - P_b)^k - (1 - P_b)^{N-1})}{(N - 1)P_b}. \tag{8}$$

Proof: The prerequisite for a successful lightpath setup over link δ from node A is that all the links along the path other than δ to be idle. Only path that is longer than k will pass traffic through link δ. Because each link is busy with probability P_b, the probability of successful lightpath setup from A can be approximated as $(1 - P_b)^m$ where $m + 1$ is the number of hops from node A to the destination. The maximal hop is $(N - 1)$ in that a node will never setup a lightpath to loop back. The traffic flow β from a node to any other node is identical, and therefore

$$\beta = \lambda/(N - 1). \tag{9}$$

Thus, the overall traffic flow from node A on link δ is approximately

$$\sum_{m=k}^{N-2} \beta(1 - P_b)^m = \frac{\lambda((1 - P_b)^k - (1 - P_b)^{N-1})}{(N - 1)P_b}. \tag{10}$$

When a link is busy, the link after this busy link is likely to be busy because the traffic flows clockwise. Due to this dependency on link blockage, the probability that k links are idle will be greater than $(1 - P_b)^k$. ■

Corollary 1: From Theorem 1, the overall traffic flow Λ on a link in the ring topology given homogeneous traffic demand satisfies

$$\Lambda = \sum_{k=0}^{N-2} I(k) \geq \frac{\lambda(1 - (1 - P_b)^{N-1}(1 + P_b(N - 1)))}{(N - 1)P_b^2}. \tag{11}$$

Theorem 2: We assume the arrival process is Poisson and the departure process is exponential with mean service time $1/\mu$. The lower bound of link blocking probability P_b can be computed by substituting Λ with the lower bound of equation (11) and by defining $\rho = \Lambda/\mu$ to solve the equation

$$\frac{\rho}{1 + \rho} = P_b. \tag{12}$$

Proof: The blocking probability on a particular link δ can be approximately computed according to M/M/1/1 as

$$\frac{\rho}{1 + \rho} = P_b^{'}. \tag{13}$$

Given P_b, $P_b^{'} \leq P_b$ because the left side of (13) is a monotonic increasing function, and we substituted ρ with its lower bound. However, P_b is also unknown. Because ρ is a monotonic decreasing function of P_b, (13) is also a monotonic decreasing function of P_b. Therefore, when $P_b^{'} \leq P_b$, the solution of P_b to (12) is strictly smaller than the precise value of P_b. ■

We denote the calculated result from (12) as P_{bl}.

V. Upper Bound of Blocking Probability

When all links are idle, the flow I_{s0} on l_1 is the maximum.

Theorem 3: In a ring with N nodes,

$$I_{s0} = N\lambda/2. \tag{14}$$

Proof: If a node A is k hops away from the head node of l_1, the probability that A imposes traffic flow on l_1 will be the left hand side of (10) with $P_b = 0$. Therefore, The total traffic flow on l_1 will be

$$\sum_{k=0}^{N-2} \sum_{m=k}^{N-2} \frac{\lambda}{N-1} = \frac{N\lambda}{2}. \tag{15}$$

■

The upper bound of link probability can never exceed

$$P_b^U = \frac{N\lambda/2}{1 + N\lambda/2} = P_{bu}^{S0}. \tag{16}$$

Equation (16) gives the simplest approach to calculate the upper bound of link blocking probability. However, we can find tighter upper bound by the following steps.

A. Lower Bound of p_{S2}

According to (6), p_{s2}=P$\{l_N$ busy and l_2 busy$\}$. Therefore $p_{s2} \geq P\{l_N \text{ busy}\}P\{l_2 \text{ busy}\} = P_b^2 \geq P_{bl}^2$. The lower bound of p_{S2} can be calculated as $p_{S2}^l = P_{bl}^2$.

B. Lower Bound of $P(S_1 \cup S_2)$

We know that $S_1 \cup S_2 = \{l_N \text{ busy}\}$ and $S_1 \cap S_2 = \Phi$. Therefore, $P_{S1} + P_{S2} = P_b \geq P_{bl}$. The lower bound of $P(S1 \cup S2)$ is P_{bl}.

C. Upper Bound of Link Blocking Probability

In S_2, the flow rate on l_1 cannot exceed $\lambda/(N-1)$. This is because only the traffic flow from the head node of l_1 to the tail node of l_1 can be accepted. Therefore, we have

$$P_{bu}^{S2} = \frac{\lambda/(N-1)}{1 + \lambda/(N-1)}. \tag{17}$$

In S_1, the flow rate on l_1 cannot exceed λ. Therefore, we have

$$P_{bu}^{S1} = \frac{\lambda}{1 + \lambda}. \tag{18}$$

Because $p_{S0} + p_{S1} + p_{S2} = 1$, we have

$$p_{S0} \leq 1 - P_{bl}. \tag{19}$$

Theorem 4: The upper bound of link blocking probability P_{bu} can be calculated as below:

$$P_{bu} = (1 - P_{bl})P_{bu}^{S0} + (P_{bl} - P_{bl}^2)P_{bu}^{S1} + P_{bl}^2 P_{bu}^{S2}. \tag{20}$$

TABLE II

Caculated Bounds in a Six-Node Ring

λ	0.001	0.0025	0.005	0.0075	0.01
P_{bl}	0.002967	0.007301	0.01423	0.02084	0.02714
P_{bu}	0.002985	0.007408	0.01464	0.02171	0.02861
P_{bu}^{S0}	0.002991	0.007444	0.01478	0.02200	0.02913

TABLE III

Caculated versus Simulated Bounds in a Three-Node Ring

λ	0.1	0.2	0.3	0.4	0.5	0.6
P_{bl}	0.1212	0.2056	0.2696	0.3206	0.3625	0.3980
P_{bc}	0.1237	0.2113	0.2776	0.3301	0.3730	0.4090
P_{bu}	0.1250	0.2144	0.2816	0.3341	0.3765	0.4116

Proof: S_0, S_1, and S_2 is a partition of the set of all network states. Therefore, from (3), we have

$$P_b < \sum_{k \in S_0} p_k P_{bu}^{S0} + \sum_{k \in S_1} p_k P_{bu}^{S1} + \sum_{k \in S_2} p_k P_{bu}^{S2} = \sum_{k=0}^{2} p_{sk} P_{bu}^{Sk}. \tag{21}$$

From (19) and $P_{bu}^{S0} > P_{bu}^{S1}$, we have

$$\sum_{k=0}^{2} p_{sk} P_{bu}^{Sk} < (1 - P_{bl})P_{bu}^{S0} + (P_{bl} - p_{S2})P_{bu}^{S1} + p_{S2} P_{bu}^{S2}. \tag{22}$$

Because $p_{S2} \geq P_{bl}^2$, we have

$$P_b < (1 - P_{bl})P_{bu}^{S0} + (P_{bl} - P_{bl}^2)P_{bu}^{S1} + P_{bl}^2 P_{bu}^{S2} = P_{bu}, \tag{23}$$

where P_{bl} can be calculated according to (12). ■

VI. Simulation versus Calculation Results

In our simulation, we assume that we are dealing with a unidirectional WDM network with 10 wavelengths in a fiber as shown in Fig. 1. The arrival of incoming call to each node is a Poisson process at a rate of 10λ per second. The distribution of the service rate is exponential with the mean of one per second. The traffic demand matrix is homogeneous.

This model is to calculate the upper bound and lower bound of blocking probability when the discrepancy between carried load and offered load is not negligible. When the traffic load is light, P_{bu}^{S0} can sever as a good upper bound. Table II provides the comparison of calculated lower bound P_{bl}, calculated upper bound P_{bu} and calculated P_{bu}^{S0} in a six-node ring.

TABLE IV

Caculated versus Simulated Bounds in a Four-Node Ring

λ	0.1	0.2	0.3	0.4	0.5	0.6
P_{bl}	0.1412	0.2249	0.2841	0.3297	0.3665	0.3973
P_{bs}	0.1543	0.2483	0.3108	0.3604	0.4001	0.4335
P_{bu}	0.1548	0.2537	0.3227	0.3738	0.4133	0.4448

TABLE V

Caculated versus Simulated Bounds in a Five-Node Ring

λ	0.1	0.2	0.3	0.4	0.5	0.6
P_{bl}	0.1537	0.2336	0.2879	0.3289	0.3619	0.3894
P_{bs}	0.1806	0.2785	0.3412	0.3895	0.4255	0.4517
P_{bu}	0.1817	0.2879	0.3583	0.4084	0.4460	0.4753

TABLE VI

CACULATED VERSUS SIMULATED BOUNDS IN A SIX-NODE RING

λ	0.1	0.2	0.3	0.4	0.5	0.6
P_{bl}	0.1614	0.2367	0.2867	0.3243	0.3543	0.3794
P_{bs}	0.2002	0.3001	0.3599	0.4072	0.4420	0.4653
P_{bu}	0.2063	0.3185	0.3897	0.4390	0.4751	0.5027

TABLE VII

ERROR AS THE NUMBER OF NODES AND TRAFFIC INTENSITY INCREASE

Number of Nodes	4	5	6
% Error of P_{bl} when $\lambda = 0.1$	8.49	14.89	19.38
% Error of P_{bl} when $\lambda = 0.2$	9.42	16.12	21.13
% Error of P_{bu} when $\lambda = 0.1$	0.32	0.61	3.05
% Error of P_{bu} when $\lambda = 0.2$	2.17	3.38	6.13

Table III provides the comparison of calculated lower bounds P_{bl}, upper bounds P_{bu} and the actual blocking probability P_{bc} when λ is increased from 0.1 to 0.6 at a step of 0.1.

Table IV to Table VI provides the comparison of calculated lower bounds P_{bl}, upper bounds P_{bu}, and the blocking probability P_{bs} from simulation when λ is increased from 0.1 to 0.6 at a step of 0.1.

From the simulation, we can observe that the calculated upper bound is very tight. When the traffic intensity is modest, the error is less than 2%.

From Table VII, we can observe that the error increases along with the node number and traffic intensity.

VII. CONCLUSION

This paper presents a novel and efficient approach to calculate the lower and upper bounds of interconnecting link blocking probability in optically switched Ring networks deploying optical network elements like ROADMs. The approach considers the impact of blocking on flow rate on a link by dividing the network state space into three subspaces. This novel approach has the potential to be applied to heterogeneous traffic demand matrix in ring or mesh networks. The efficiency of the approach provides the network designers a useful tool to control the QoS or adjust the traffic flow in their networks

REFERENCES

[1] R. Ramaswami and K. Sivarajan, *Optical Networks: A Practical Perspective*, 2nd ed., ser. Networking. Morgan Kaufmann, 2002.

[2] E. Mannie and Ed., "Generalized multi-protocol label switching (GMPLS) architecture," *RFC3945*.

[3] R. A. Barry and P. A. Humblet, "Models of blocking probability in all-optical networks with and without wavelength changers," *IEEE Journal on Selected Areas in Communications*, vol. 14, no. 5, pp. 858–867, June 1996.

[4] L. wei Chen and E. Modiano, "Efficient routing and wavelength assignment for reconfigurable wdm ring networks with wavelength converters," *IEEE/ACM Transactions on Networking*, vol. 13, no. 1, pp. 173–186, Feb. 2005.

[5] S. Subramaniam, M. Azizogelu, and A. K. Somani, "All-optical networks with sparse wavelength conversion," *IEEE/ACM Transactions on Networking*, vol. 4, no. 4, pp. 544–557, Aug. 1996.

[6] Y. Luo and N. Ansari, "A computational model for estimating blocking probabilities of multifiber wdm optical networks," *IEEE Communications Letters*, vol. 8, no. 1, pp. 60–62, Jan. 2004.